Oral Exam Syllabus
Jawon Koo
2006

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I. Financial Mathematics

i. Probability Theory

a. Conditional expectation

b. Martingale inequalities: Doob's maximal inequality,Doob's stopping time theorem

c. Martingale convergence theorem: L^2convergence and a.e convergence.

ii. Stochastic Process

a. Brownian motion: definition, path property Lévy's characterization,Strong Markov property

b. Poisson process: Poisson random measure,Compound Poisson process

c. Lévy process: Lévy-Itô decomposition, Lévy-Khintchine formula

iii. Stochastic calculus

a. Itô integral

b. Itô-Doeblin Formula

c. Lévy type stochastic integral

d. Itô formula for Lévy process

e. Girsanov's theorem

f. Martingale representation theorem

iv. Risk-Neutral European Option Pricing

a. Pricing European options in geometric Brownian model, Deriving closed form of BSM formula

b. Pricing European options in exponential Lévy model, Fourier transform methods for option pricing by Carr and Madan

c. Fundamental Theorems of Asset Pricing

v. Connection with Partial Differential Equation

a. Feynman-Kac Theorem
b. Derivation of BSM PDE
c. Derivation of Partial integro-differential equations for computing option prices

II. Partial Differential Equation

i. Heat Equation

a. Representation of its solution in the unbounded domain:Derivation of Fundamental solution and its role.
b. Representation of its solution in the bounded domain(IBP):series solution,Fundamental solutions.
c. Weak and Strong Maximum Principle and Energy estimates, uniqueness of solutions,Estimation of solutions
d. Green's identity and smoothness of solutions.
e. Gradient & higher derivative estimates, sequences of solutions and its limit.
f. Numerical method : finite difference and its stability,convergence.

ii. Second order parabolic equations

a. Equidimensional or constant coefficients case : Change of variables,Fourier transform
b. Bounded variable coefficients case for initial boundary value problem: definition of weak solution
c. Existence of weak solution : Galerkin approximation
d. Maximum principle and uniqueness of solutions

References

i. Shreve, Steven E., *Stochastic Calculus for Finance II*
ii. Steele, Michael J., *Stochastic Calculus and Financial Applications*
iii. Cont, Rama *Financial Modelling with Jump Processes*
iv. Applebaum, David *Levy Processes and Stochastic Calculus*
v. Evans, Lawrence C., *Partial Differential Equation*
vi. Fritz, John *Partial Differential Equation*