Oral Qualifying Exam Syllabus

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1 Combinatorics

Basic Enumeration: counting arguments, binomial coefficients, recurrence relations and generating functions, inclusion-exclusion principle, Stirling's rule
Set Systems: Sperner lemma, LYM inequality, Erdős-Ko-Rado, Kruskal-Katona
Algebraic Methods: Combinatorial Nullstellensatz, Chevaley-Warning, Ray-Chaudhuri-Wilson, Frankl-Wilson
Correlation Inequalities: Harris-Kleitman, Fortuin-Kasteleyn-Ginibre (FKG), Ahlswede-Daykin four functions, application to Shepp's XYZ theorems
Ramsey Theory: Ramsey's theorem, infinite Ramsey, König's lemma, probabilistic lower bounds, van der Waerden, statement of Szemerédi's theorem

2 Graph Theory

Matching: König, Hall, Tutte's 1-factor
Connectivity: structure of 2-connected graphs, structure of 3-connected graphs, Menger's theorem, Max Flow-Min Cut
Planarity: Euler's formula, Kuratowski's theorem, Warner's theorem
Coloring: 5 color theorem, Brooks, Vizing, Thomassen's 5-list-coloring of planar graphs, Galvin's proof of Dinitz conjecture, Lovász's proof of weak perfect graph conjecture
Extremal Problems: Turán, Erdős-Stone, statement of regularity lemma

3 Probabilistic Methods

Basics: linearity of expectation, alterations, Chebyschev's inequality, Chernoff bound
Second Moment Method: general procedure, application to threshold function for having a certain graph as a subgraph
Lovász Local Lemma: symmetric and general versions, applications to Latin transversals

Martingales: Azuma's inequality, edge and vertex exposure, application to chromatic number
Poisson Paradigm: Janson inequalities, Brun's sieve, application to number of triangles in $G_{n,p}$
Random Graphs: monotone properties, existence of threshold functions, clique number, chromatic number (Bollobás)

4 Additive Number Theory

Structure of sumsets and applications: basic definition and results, Rusza distance and additive energy, statement of Balog-Szemeredi-Gowers theorem.
Geometry of numbers: basic results for lattices and convex bodies in $\mathbb{R}^d$, John's theorem, Rusza Covering lemma, Volume Packing lemma, Blichfeldt's lemma, Minkowski's two theorems, Discrete John's theorem.
Generalized Arithmetic Progressions (GAPs): definition of proper and non-proper GAPs, containing lemmas.
Freiman's theorem: Freiman's isomorphism, Freiman's Cube lemma, Rusza's lemma, Freiman's theorem.
Discrete Fourier Analysis and the Littlewood-Offord problem: definition $X_v^{(\mu)}$ and basic properties of its Fourier representation, Halász-type concentration inequality, basic Littlewood-Offord results, stronger Littlewood-Offord result with Fourier analysis.