**Oral Qualifying Exam Syllabus**

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## 1 Combinatorics

**Basic Enumeration:** counting arguments, binomial coefficients, recurrence relations and generating functions, inclusion-exclusion principle, Stirling's rule
**Set Systems:** Sperner lemma, LYM inequality, Erdős-Ko-Rado, Kruskal-Katona
**Algebraic Methods:** Combinatorial Nullstellensatz, Chevaley-Warning, Ray-Chaudhuri-Wilson, Frankl-Wilson
**Correlation Inequalities:** Harris-Kleitman, Fortuin-Kasteleyn-Ginibre (FKG), Ahlswede-Daykin four functions, application to Shepp's XYZ theorems
**Ramsey Theory:** Ramsey's theorem, infinite Ramsey, König's lemma, probabilistic lower bounds, van der Waerden, statement of Szemerédi's theorem

## 2 Graph Theory

**Matching:** König, Hall, Tutte's 1-factor
**Connectivity:** structure of 2-connected graphs, structure of 3-connected graphs, Menger's theorem, Max Flow-Min Cut
**Planarity:** Euler's formula, Kuratowski's theorem, Warner's theorem
**Coloring:** 5 color theorem, Brooks, Vizing, Thomassen's 5-list-coloring of planar graphs, Galvin's proof of Dinitz conjecture, Lovász's proof of weak perfect graph conjecture
**Extremal Problems:** Turán, Erdős-Stone, statement of regularity lemma

## 3 Probabilistic Methods

**Basics:** linearity of expectation, alterations, Chebyschev's inequality, Chernoff bound
**Second Moment Method:** general procedure, application to threshold function for having a certain graph as a subgraph
**Lovász Local Lemma:** symmetric and general versions, applications to Latin transversals

**Martingales:** Azuma's inequality, edge and vertex exposure, application to chromatic number
**Poisson Paradigm:** Janson inequalities, Brun's sieve, application to number of triangles in $G_{n,p}$
**Random Graphs:** monotone properties, existence of threshold functions, clique number, chromatic number (Bollobás)

## 4 Additive Number Theory

**Structure of sumsets and applications:** basic definition and results, Rusza distance and additive energy, statement of Balog-Szemeredi-Gowers theorem.
**Geometry of numbers:** basic results for lattices and convex bodies in $\mathbb{R}^d$, John's theorem, Rusza Covering lemma, Volume Packing lemma, Blichfeldt's lemma, Minkowski's two theorems, Discrete John's theorem.
**Generalized Arithmetic Progressions (GAPs):** definition of proper and non-proper GAPs, containing lemmas.
**Freiman's theorem:** Freiman's isomorphism, Freiman's Cube lemma, Rusza's lemma, Freiman's theorem.
**Discrete Fourier Analysis and the Littlewood-Offord problem:** definition $X_v^{(\mu)}$ and basic properties of its Fourier representation, Halász-type concentration inequality, basic Littlewood-Offord results, stronger Littlewood-Offord result with Fourier analysis.