

# Oral Exam Syllabus

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## **1 Functional Analysis**

### **1.1 Topological Vector Spaces**

- A. Basic Properties and Examples
- B. Locally Convex Spaces
- C. Banach Spaces

### **1.2 Baire Category Theorem and Consequences**

- A. Baire Category
- B. Uniform Boundedness Principle
- C. Open Mapping Theorem
- D. Closed Graph Theorem

### **1.3 Duality and Convexity**

- A. Hahn-Banach Theorems
- B. Banach-Alaoglu Theorem
- C. Krein-Milman Theorem

### **1.4 Fixed Point Theorems and Applications**

- A. Basic Fixed Point Theorems (Brouwer and Contraction Principle)
- B. Markov-Kakutani
- C. Local Solutions of ODE

## 1.5 Hilbert Spaces

- A. Geometry of Hilbert Space
- B. The Algebra of Operators
- C. Spectral Theorem for Hermitian and Normal Operators
- D. Unbounded Operators

## 2 Harmonic Analysis

### 2.1 Fourier Transform

- A. Construction and Basic Properties
- B. Application to  $L^2$
- C. Schwartz Space
- D. Tempered Distributions
- E. Operators that Commute with Translations

### 2.2 Some Fundamental Notions

- A. The Hardy-Littlewood Maximal Function
- B. Lebesgue Differentiation Theorem
- C. Vitali Covering Lemma
- D. Marcinkiewicz Integral
- E. Calderon-Zygmund Decomposition

### 2.3 Interpolation of Operators

- A. Riesz-Thorin
- B. Marcinkiewicz Interpolation Theorem

### 2.4 Singular Integrals

- A.  $L^p$  Boundedness
- B. Hilbert Transform
- C. Riesz Transform
- D. Applications

Primary References:

Rudin, *Functional Analysis*

Stein, *Singular Integrals and Differentiability Properties of Functions*

Stein and Weiss, *Introduction to Fourier Analysis on Euclidean Spaces*