Topics for oral qualifying exam for Liang Kong
Fall, 2001

Major topic: Vertex operator algebras

The general theory of vertex operator algebras, basic examples, the basic geometry of vertex operator algebras as presented in [FLM] (Chapters 1–8), [FHL], [H] and [L]. The specific topics are:

1. Definitions and properties.
   (a) Formal calculus.
   (b) The notions of vertex algebra and of vertex operator algebra, and basic properties.
   (c) Rationality, commutativity and associativity (various formulations, including "weak" formulations.
2. Examples of vertex (operator) algebras.
   (a) Vertex (operator) algebras based on Heisenberg Lie algebras.
   (b) Vertex (operator) algebras based on affine Lie algebras.
   (c) Vertex (operator) algebras on the Virasoro algebra.
   (d) Vertex (operator) algebras on even lattices.
3. Representations of vertex (operator) algebras.
   (a) The notion of module and basic properties.
   (b) The relations between this notion and the topics listed above, including the construction of modules from mutually local operators.
   (c) The notions of intertwining operator and fusion rule, basic properties.
   (d) Examples of modules for the vertex operator algebras listed above.
4. The geometry of vertex operator algebras.

(a) The moduli spaces of spheres with tubes, the sewing operation, examples for the sewing operation.

(b) The geometric interpretations of vertex operators and vacua, the geometric meanings of commutativity, associativity, skew-symmetry, $L(-1)$-derivative property, vacuum property and creation property.

(c) The notion of geometric vertex operator algebra and the statement of the isomorphism theorem.

(d) Determinant lines of Fredholm operators, determinant lines over Riemann surfaces with parametrized boundaries, relation to the central charges of vertex operator algebras.

Minor topic: Quantum field theory

Basic quantum field theory as presented in [IZ] and [PS]. The specific topics are:

1. Classical field theory
   - (a) Action, Lagrangian, action principle and Euler-Lagrange Equation.
   - (b) Canonical momentum, Hamiltonian.
   - (c) Symmetry, Noether current, stress-energy tensor.
2. Canonical quantization of free theories.
3. Interaction quantum theory.
   - (a) n-point Green's functions, Wick's theorem.
   - (b) Feynman diagrams.
   - (c) Loop integral computation, dimensional regularization.
4. Functional integral formulation of quantum field theory.
   - (a) Partition function, Feynman-Kac formula.
   - (b) Feynman diagram.
   - (c) Equation of motion, Ward's identity.

5. Systematic renormalization.

   (a) Renormalizability, counterterms.
   (b) 1-loop result of the ϕ^4 theory.
   (c) Callan-Symanzyk equation.
   (d) Running coupling constant, $\beta(g)$, and its one-loop calculation. (optional)

References

[FHL] I. Frenkel, Y.-Z. Huang and J. Lepowsky, On Axiomatic Approaches to Vertex Operator Algebras and Modules, Memoirs Amer. Math. Soc. 104 (1993).

[FLM] I. Frenkel, J. Lepowsky and A. Meurman, Vertex Operator Algebras and the Monster, Academic Press, 1988.

[H] Y.-Z. Huang, Two-dimensional conformal geometry and vertex operator algebras, Progress in Math., Vol. 148, Birkhäuser Boston, 1997.

[L] H. Li, Local systems of vertex operators, vertex superalgebras and modules, J. Pure and Applied Algebra 109 (1996), 143-195.

[IZ] C. Itzykson and J. B. Zuber, Quantum field theory, International Series in Pure and Applied Physics. McGraw-Hill International Book Co., New York, 1980.

[PS] M. E. Peskin and D. V. Schroeder, An introduction to quantum field theory, edited and with a foreword by D. Pines, Addison-Wesley Publishing Company, Advanced Book Program, Reading, MA, 1995.