

Oral Exam Syllabus

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1 Major topic: Mathematical Biology

1.1 Continuous models for population dynamics

- Models for single species populations: Malthus law, logistic growth
- Predator-Prey systems and Lotka-Volterra equations
- Populations in competition

1.2 Models for molecular events

- Basic mass action
- Enzyme kinetics: Michaelis-Menten, Burk plots, allosteric inhibitors, competitive inhibition and cooperativity (Hill equation)
- Quasi-steady-state analysis, time-scale separation
- Biological oscillators and switches
- Activator-Inhibitor and positive feedback systems
- Probabilistic channel model

1.3 Partial differential equation models in biology

- Diffusion: the derivation of diffusion equation, speed of diffusion
- Diffusion through a membrane: Ohm's Law
- Facilitated diffusion
- The membrane potential: Nernst equation

1.4 Excitable systems, limit cycles and oscillations

- Excitable systems: Hodgkin-Huxley model, Fitzhugh's analysis of Hodgkin-Huxley equations
- periodic orbits and limit cycles
- Poincare-Bendixon theory
- Hopf bifurcation

1.5 Development and Pattern Formation

- Conditions for diffusive instability
- Turing's approach to morphogenesis

2 Minor topic: Partial Differential Equations

2.1 Laplace equations

- Derivation of the fundamental solution
- Mean value formulas
- Properties of harmonic function: maximum principle, smoothness, local estimates on derivatives, Harnack inequality

2.2 Second order elliptic equations

- Second order elliptic operator
- Weak maximum principle
- Hopf Lemma
- Strong maximum principle

2.3 Sobolev spaces

- Weak derivatives
- Sobolev spaces
- Approximation: interior/global approximation by smooth functions
- Gagliardo-Nirenberg-Sobolev inequality
- Poincare inequality

References

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- [3] J.D. Murray, *Mathematical Biology*, Springer-Verlag, 1993.
- [4] L.C. Evans, *Partial Differential Equations*, AMS Providence, 1998.
- [5] D. Gilbarg and N.S. Trudinger, *Elliptic Partial Differential Equations of Second Order*, Springer-Verlag, 1983.