

Oral Qualifying Examination Syllabus

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Committee(in alphabetical order): R. Bumby, H. Iwaniec, J. Tunnell.

1. Algebraic Number Theory

- (a) Invariants of number fields: rings of integers, discriminants and orders
- (b) Arithmetic of number fields: splitting of primes, ramification, Frobenius
- (c) Class groups
- (d) Structure of units in number rings
- (e) Binary quadratic forms

2. Analytic Number Theory

- (a) Elementary estimates of standard arithmetic functions
- (b) Analytic properties of L -functions and the Riemann zeta functions
- (c) Primes in arithmetic progression
- (d) Siegel zero problem
- (e) Prime number theorem and Prime Number Theorem for Arithmetic Progressions
- (f) The large sieve inequality for multiplicative characters

3. Elliptic Curves

- (a) Elliptic curves over the complex field: Elliptic functions and the j -function
- (b) Elliptic curves over finite fields
- (c) Hasse-Weil L -functions of elliptic curves
- (d) Mordell-Weil Theorem and descent on elliptic curves

4. Modular Forms

- (a) Modular Forms for the full modular group and its congruence subgroups
- (b) Eisenstein series
- (c) Structure of the ring of modular forms
- (d) Mellin transforms and Dirichlet L -series
- (e) Hecke operators
- (f) Modular forms of half integer weight

References

- [JC] J. Cassels, Lectures on Elliptic Curves, Cambridge, 1991,
- [HD] H. Davenport, Multiplicative Number Theory, Springer-Verlag, 2000.
- [HI] H. Iwaniec, Course Notes.
- [NK] N. Koblitz, Introduction to Elliptic Curves and Modular Forms, Springer, 1993.
- [DM] D. Marcus, Number Fields, Springer-Verlag, 1997.
- [HM] H. Montgomery, Topics in Multiplicative Number Theory, Springer-Verlag, 1997.
- [JT] J. Tunnell, course notes, Fall, 2000.