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Major : Topological methods In critical point theory,

A Priori Estimates for Elliptic Operators: [Br],[GT]

- Sobolev imbedding
- $W^{k,p}$ Estimates
- $C^{k,\alpha}$ Estimates.

Degree theory,[Ch]

- construction in finite dimension
- infinite dimensional case
- applications

Calculus of variation, [Ch]

- Direct method
- example

Deformation Lemmas [Ch], [Ch2]

- pseudo-gradient
- first deformation lemma,
- second deformation Lemma,

Minmax Principle [Ch], [Ch2]

- Examples

Lusternik-Schnirelman category, [Ch], [Ch2], [Iv]

Multiplicity results [Ch], [Ch2], [Iv]

Index theory [Ch]

- genus
- S^1 -index

Critical groups [Ch], [Ch2]

- Critical groups at a point
- critical groups at infinity
- Morse type results.
- Connection with the degree
- examples

Minor : Morse Theory,

Morse Lemma, [Mi]

Existence of Morse Functions, [Mi]

Gradient Flow, [Mi]

Deformation lemma, [Mi], [Ch]

Morse homology : Attaching cells, [Mi]

Definition of Stable and Unstable And orientation convention, [Ba]

Morse Smale,

- Transversality condition, [Ba],[ju],[Ke]
- Moduli space of trajectories, [ju], [Ke]
- compactification [Ke]
- boundary operator, [ju]
- Construction of the homology [ju]

Infinite dimension extension :

- Geodesics problem and Homology of the path space. [Mi]
 - First Variation
 - Second variation
 - Index and Existence of conjugate points
- Overview on Floer Homology, [Mi]
 - Action functional and J-holomorphic curves
- Application on contact geometry, [Bo]
 - Reeb vector field,
 - Action functional and Periodic orbits

References

- [Ba] A. Banyaga, D. Hurtubise, Lectures On Morse Homology, Kluwer Academic Publishers 2004
- [Bo] F. Bourgeois, Introduction to Contact Homology, Summer School in Berder : Holomorphic curves and contact topology 2003.
- [Br] H. Brézis, Analyse Fonctionnelle théorie et application, Masson (1983).
- [Ch2] Kung-Ching Chang, Infinite dimensional morse theory And Multiple solution Problems,
- [Ch] Kung-Ching Chang, Methods in nonlinear analysis, Springer 2005.
- [Iv] I. Ekeland, A. Szulkin, Minimax Results of Lusternik-schnirelman type And applications, Séminaire de Mathématique Supérieures 1989.
- [GT] D. Gilbarg, N.S. Trudinger, Elliptic Partial Differential Equation of Second Order, Springer-Verlag (1977)
- [Go] C. Godbillon, Géométrie Différentielle et Mécanique Analytique, Hermann 1969.
- [Ke] E. Kerman, From Morse Homology to Floer Homology, informal lecture notes for MATH 621.
- [Mi] J. Milnor, Morse Theory, Princeton University Press 1963..
- [ju] Juanita Pinzon Caicedo, Morse Homology, Thesis Universidad de Los Andes 2007.