Oral Qualifying Exam Syllabus

Brian Manning

MAJOR TOPIC: RIEMANNIAN GEOMETRY

Preparation: Manifolds, vector bundles, tensors, differential forms
Riemannian metrics
Connections, covariant derivative, parallel transport
Exponential map, geodesics, Gauss Lemma
Convex neighborhoods
Completeness, geodesic completeness; Hopf-Rinow Theorem
Sectional Curvature, Ricci Curvature, and Scalar Curvature
First and second variation formulas
Jacobi Fields
Submanifolds and immersions
Distance functions, curvature equations
Classification of constant curvature spaces
First and second comparison estimates
Hadamard-Cartan theorem, Bonnet-Myers theorem
Conjugate points, conjugate radius, injectivity radius
Fundamental group and nonpositive curvature
Hypersurfaces in Riemannian manifolds
Synge's theorem
Lie Groups, bi-invariant metrics
Free isometric group action
Riemann submersion formula, homogeneous spaces

References: Karsten Grove, Riemannian Geometry: A Metric Entrance
Peter Petersen, Riemannian Geometry

MINOR TOPIC: ALGEBRAIC TOPOLOGY

Covering spaces, path lifting
Homotopy, homotopy lifting theorem
Fundamental group, Van Kampen Theorem
Singular homology group, chain complexes, Homotopy invariance
Relative homology
Exact sequences, Excision theorem, Mayer-Vietoris sequences
Betti numbers and Euler characteristics
Orientation of a manifold
Singular cohomology, Cup and cap products

References: Allen Hatcher, Algebraic Topology
James Vick, Homology Theory: An Introduction to Algebraic Topology