

Oral Qualifying Exam Syllabus

Michael Marcondes de Freitas

February 23, 2010

Committee

- Daniel Ocone
- Roger Nussbaum
- Eduardo Sontag
- Eugene Speer

Major Topic: Ordinary Differential Equations and Monotone Systems

Basic Ordinary Differential Equations Theory

- Existence, uniqueness and continuous dependence on initial conditions and parameters
- Orbits and asymptotic limit sets
- Poincaré-Bendixson Theorem

Monotone Dynamical Systems

- The convergence criterion
- The limit set dichotomy
- Quasiconvergence results

Stability and Convergence

- Stability
- The order interval trichotomy
- Global results

- Generic convergence to equilibrium
- Unstable equilibrium and connecting orbits

Competitive and Cooperative Differential Equations

- The Kamke condition
- Positively invariant sets and monotone solutions
- Three-dimensional systems
- Alternative cones
- The Field-Noyes model

Irreducible Cooperative System

- Strong monotonicity
- Biochemical control circuits
- Stability and the Perron-Frobenius Theorem
- Competition and migration
- Smale's construction

Minor Topic: Mathematical Biology

Deterministic Models

- Growth models: exponential growth, logistical equation, chemostat
- Reducing number of parameters
- Steady states and linearized stability analysis
- Phase planes: vector fields, nullclines, global behavior
- Effect of drug on cells in an organ
- Compartment models
- Epidemiology models: immunization, STD models
- Chemical kinetics: chemical networks, quasi-steady state approximations, fast and slow behavior, singular perturbation analysis, inhibition, cooperativity
- Multi-stability
- Periodic Behavior
- Bifurcations

Stochastic Biochemical Models

- Stochastic models of chemical reactions
- The Chemical Master Equation
- Theoretical background and algorithms: Markov processes, interpretation of the master equation and propensity functions, the stochastic simulation algorithm, interpretation of mass-action kinetics
- Moment equations and fluctuation-dissipation formula
- Generating functions
- Conservation laws and stoichiometry

References

- [1] Smith, Hal L.; *Monotone Dynamical Systems: An Introduction to The Theory of Competitive and Cooperative Systems*. Mathematical Surveys and Monographs, Volume 41. American Mathematical Society, 1995.
- [2] Sontag, Eduardo D.; *Lecture Notes on Mathematical Systems Biology*. Rutgers University, 2009. Available for download at http://www.math.rutgers.edu/~sontag/slides_systems_biology_notes_ode_models.pdf.
- [3] Speer, Eugene R.; *Lecture Notes on Ordinary Differential Equations*. Rutgers University, 2000. Available for download at <http://www.math.rutgers.edu/courses/515/speer-notes.html>.