Ph. D. Qualifying Oral Examination
Candidate: Carlo Mazza

Homological Algebra.

1. Chain Complexes

   - Complexes of Modules.
   - Long Exact Sequences.
   - Chain Homotopies.
   - Mapping Cones and Cylinders.

2. Derived Functors

   - Derived Functors.
   - Projective and Injective Resolutions.
   - Left and Right Derived Functors
   - Tor and Ext.

3. Homological Dimension

   - Dimensions.
   - Local Rings.
   - Koszul Complexes.

4. Spectral Sequences

   - Introduction and Terminology.
   - The Leray-Serre Spectral Sequence.
   - Spectral Sequence of a Filtration.
   - Convergence.
   - Spectral Sequences of a Double Complex.
   - Hyperhomology.
   - Grothendieck Spectral Sequences.
   - Exact Couples.

5. Simplicial Methods

- Simplicial Objects.
- Dold-Kan Correspondence.
- The Eilenberg-Zilber Theorem.

Commutative Algebra & Algebraic Geometry

1. Basic Commutative Algebra

- Rings and Modules.
- Rings and Modules of Fractions.
- Primary Decomposition.
- Integral Dependence and Valuations.
- Chain Conditions.
- Noetherian Rings.
- Artin Rings.
- DVR's and Dedekind Domains.
- Completions.
- Dimension Theory.

2. Varieties

- Affine and Projective Varieties.
- Morphisms.
- Rational Maps.

3. Schemes

- Sheaves.
- Schemes.
- First Properties of Schemes.
- Separated and Proper Morphisms.
- Divisors.
- Projective Morphisms.

4. Cohomology

- Derived Functors.
- Cohomology of Sheaves.
- Cohomology of a Noetherian Affine Scheme.
- Cech Cohomology.
- The Cohomology of Projective Space.
- Ext Groups and Sheaves.
- The Serre Duality Theorem.
- Higher Direct Images of Sheaves.

References

C. A. Weibel, *An Introduction to Homological Algebra*, Cambridge University Press, 1994.

M. F. Atiyah and I. G. MacDonald, *Introduction to Commutative Algebra*, Addison-Welsey, 1969.

R. Hartshorne, *Algebraic Geometry*, Springer-Verlag, 1977.