Syllabus for Qualifying Exam, Spring 2000

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Topic 1: Control Theory

1. Linear time-invariant finite-dimensional systems
   - formula for solution
   - continuous and discrete time

2. Controllability of linear systems
   - time-invariant controllability rank condition
   - Kalman controllability decomposition
   - Hautus Lemma
   - controllability under sampling
   - computing controls; finite horizon least squares
   - time-varying controllability rank condition

3. Controllability of nonlinear systems
   - local controllability
   - accessibility rank condition
   - reversible systems

4. Outputs
   - observability of time-invariant linear systems
   - sampled observability
   - local observability

5. Feedback
   - constant linear feedback; Pole-Shifting Theorem
   - disturbance rejection and invariance
   - stability and stabilizability
   - control-Lyapunov functions
   - observers and detectability; dynamic feedback

6. Linear Quadratic control
   - LQ systems
   - Deterministic Kalman filtering
   - infinite time problems

Topic 2: Numerical Analysis

1. Polynomial Approximation
   - Lagrange interpolation

- Cubic Hermite interpolation
- Piecewise polynomial approximation
- Some error results

2. Numerical Quadrature
   - (composite) Trapezoidal, Simpson's and midpoint rules
   - Derivation and error formulas
   - Basic results of Gaussian quadrature formulas

3. Numerical methods for ordinary differential equations
   - Derivation and error estimates for one-step methods (e.g., Euler's method)
   - Multistep methods (examples of explicit and implicit methods)
   - Predictor-corrector methods
   - Consistency, stability, and convergence of multistep methods

4. Finite Difference methods for partial differential equations
   - Laplace's equation: 5 point difference scheme, error estimates using discrete maximum principle
   - Simple difference approximations of time dependent equations (transport, heat, and wave equations)
   - Error analysis by the maximum principle
   - Von Neumann stability condition

5. Finite Element methods for elliptic partial differential equations
   - Standard variational formulation of boundary value problems with Dirichlet or Neumann boundary conditions
   - Energy norm error estimates
   - Solution of the resulting matrix equations (existence and uniqueness of solutions, iterative schemes)