Syllabus for Oral Qualifying Exam

Ming Xiao

Committee: Hector Sussmann, Xiaojun Huang(Chair), Xiaochun Rong, Jian Song

I. Complex analysis in Several Complex Variables

1. Analytic Automorphism group of some special domain:

- Laurent expansion in Reinhardt domain.
- Analytic automorphism group of polydisc Δ^n.
- Analytic automorphism group of unit ball B^n, Cartan's Theorem.
- Poincaré Theorem.

2. Cauchy integral formula and its application.

- Cauchy integral formula in polydisc, cauchy estimates.
- Bochner-Matinelli formula.
- Hartogs extension Theorem.
- Bochner extension Theorem.

3. Subharmonicity and convexity:

- Properties of subharmonic functions and plurisubharmonic functions.
- Domain of holomorphy, Continuity Principle.
- Pseudoconvexity, Levi pseudoconvexity.
- Oka's Theorem.

4. L^2 theory for $\bar{\partial}$ on pseudoconvex domains:

- Morrey-Kohn-Hormander Theorem.
- L^2 Existence Theorem for $\bar{\partial}$ operator.

- $\bar{\partial}$-Neumann problem.
- The Levi problem.

II. **Some Riemannian and Complex Geometry**
   1. Riemann metric and connections.
   2. Curvature.
   3. Jacobi field
   4. Hopf-Rinow Theorem and Hamamard Theorem
   5. Kodaira-Bochner formula
   6. Kodaira vanishing theorem.
   7. Kodaira embedding theorem.
   8. Hodge manifold and embedding.
   9. Riemann-Roch Theorem on compact Riemann Surface.

References

[LH] Lars Hörmander, An Introduction to Complex Analysis in Several Complex Variables.

[M-K] Morrow, Kodaira, Complex Manifolds.

[G-H] Griffiths, Harris, Chapter 0. Principles of Algebraic Geometry.