Syllabus Oral Qualifying Exam

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I Functional Analysis

1. Hilbert spaces
   Riesz lemma, orthonormal bases, mean ergodic theorem.

2. Banach spaces
   Dual spaces, Hahn-Banach theorem, Baire category theorem, uniform boundedness principle, open mapping theorem, closed graph theorem.

3. Locally convex spaces
   Fréchet spaces, Schwartz space, tempered distributions.

4. Bounded operators
   Operator topologies, adjoint, spectrum, positive operators, polar decomposition.

5. The spectral theorem
   Functional calculus, spectral measures, spectral projections, Stone's formula.

6. Unbounded operators
   Closed operators, self-adjoint operators, spectral theorem, Stone's theorem, quadratic forms, convergence of unbounded operators.

II C*-algebras

1. Spectral theory and Banach algebras

2. Operators on Hilbert space
   Commutative C-algebras, the spectral theorem for normal operators, compact operators.*

3. Compact perturbations and Fredholm theory
   Fredholm alternative, Fredholm operators.

4. States and the GNS construction
   The Gelfand-Naimark theorem.

References

(1) M. Reed, B. Simon, *Methods of modern mathematical physics. Vol. I. Functional analysis.* Academic Press, 1980.

(2) W. Arveson, *A short course on spectral theory.* Springer-Verlag New York, Inc., 2002.