Topics for oral qualifying exam: Moulik Kallupalam Balasubramanian
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Major topic: Non-linear wave equation

1. Background
   (a) Representation formulae for u in $\mathbb{R}^{1+n}$ in the linear problem:$(\partial_t^2 - \Delta)u(t,x) = F(t,x)$.
   (b) Energy inequality, when second order coefficients are close to D'Alembertian.
   (c) Uniqueness theorem of John for C^2 solution of $\Box u = F(u, u', u'')$,
   (d) Local(in time) existence, uniqueness for weak solutions on $\mathbb{R}^{1+n}$ of $Lu = F$, where L is linear and close to D'Alembertian.
   (e) Local(in time) existence on $\mathbb{R}^n$ for quasilinear equation $\Sigma g^{jk}(u,u')\partial_j\partial_k u = F(u,u')$close to the D'Alembertian, characterization of finite maximal existence time.
   (f) Local(in time) existence for semi-linear equation $\Box u(t,x) = F(u(t,x)$ on $\mathbb{R}^{1+3}$-and sup norm blow at finite maximal existence time.
2. Quasilinear Hyperbolic Equations with Small Data
   (a) Klainerman Sobolev Inequalities
   (b) Global existence on $\mathbb{R}^{1+n}$ with $n \geq 4$ to $\Sigma_{j,k=0}g^{jk}(u')\partial_j\partial_k u = F(u'); u(0,.) = \epsilon f(.); (\partial_t u)(0,.) = \epsilon g(.)$ with for ϵ small, when the linearization is the linear homogenous wave equation.
   (c) Almost global existence in dimension 3(John and Klainerman)
   (d) Almost global existence for symmetric non-linear system in $\mathbb{R}^{1+3}$
   (e) Global existence for symmetric systems on R^3 when null condition is assumed.
3. Strichartz estimates and applications
   (a) Original Strichartz Estimate on $\mathbb{R}^3$
   (b) Application- global existence for semi-linear equations of the kind $\Box u = u^k$ with $k \geq 3$ in $\mathbb{R}^{1+3}$.
   (c) Endpoint Strichartz estimate of Keel and Tao and application to wave equation.

Minor Topics 1: Functional Analysis and PDE

1. Functional Analysis
   (a) Hahn Banach Theorem
   (b) Baire Category theorem and consequences
   (c) Riesz Thorin and Marcinkiewicz interpolation.
   (d) Distributions, Schwartz functions, tempered distributions.
   (e) $W^{k,p}$ spaces, Sobolev Embedding Theorem, Rellich-Kondrachev Compactness
   (f) Fourier Transform.
   (g) Sobolev space $H^s(\mathbb{R}^n)$ using Fourier Transform.
   (h) Elliptic Regularity for Constant Coefficient Linear operators.

2. Laplace's Equation
    (a) Green's representation.
    (b) Mean value inequalities, Maximum Principle and Harnack.
    (c) Poisson Kernel
    (d) Interior derivative estimates
    (e) Perron's method.

Minor topic 2: Geometry and Physics

1. Geometry
    (a) Semi-Riemannian metric
    (b) Levi-Civita Connection
    (c) First and second variation of arc length and energy.
    (d) Geodesics,completeness, Hopf-Rinow theorem.
    (e) Parallel transport, curvature tensors.
    (f) Submanifolds-second fundamental form, Gauss equation, Codazzi equation.
2. Mathematical Physics
    (a) Lagrangian Framework
    (b) Conservation Laws
    (c) Scalar Field Equations
    (d) Linear Maxwell Equations
    (e) Maxwell-Born-Infeld Equations
    (f) Einstein's equations
    (g) Gauge Invariance in Maxwell's and Einstein's equations
    (h) Minkowski space and its features
    (i) Schwarzchild metric and its features.
    (j) Basic Causality Concepts
    (k) Penrose Diagrams

1 References

1. Christopher D.Sogge, Lectures on Non-Linear Wave Equations.
2. Shatah and Struwe, Geometric Wave Equations
3. Keel and Tao, Endpoint Strichartz Estimates(in American Journal of Mathematics)
4. Folland, Real Analysis.
5. Gallot Hulin and Lafontaine, Riemannian Geometry
6. O'Neil, Semi-Riemannian Geometry.
7. Gilbarg and Trudinger, Elliptic Partial Differential Equations of Second Order.