Oral qualifying exam syllabus

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1 Algebraic combinatorics

- **Coxeter groups:** Definition, length function, Bruhat order, weak order, root systems/geometric representation, reduced words, realization of classical finite and affine Weyl groups as permutation groups
- **Partially ordered sets:** Hasse diagrams, lattices, Birkhoff representation theorem, incidence algebras, Möbius functions, linear extensions as maximal chains and lattice paths, Jordan-Hölder set
- **Tableaux:** Robinson-Schensted-Knuth correspondence, Knuth relations/jeu de taquin (for shifted tableaux as well, as per Worley), Greene invariant, promotion and evacuation, Edelman-Greene correspondence (especially for Grassmannian elements), Schur functions
- **Schubert calculus:** Bruhat decomposition, parabolic subgroups, flag varieties, Schubert cells/varieties, Schubert classes form a basis for cohomology, invariant polynomials of finite reflection groups and connection to cohomology of flag varieties

2 Algebraic geometry

- **Sheaves:** Definition, presheaves and associated sheaves, morphisms, stalks
- **Schemes:** Affine schemes, Proj, structure sheaf, scheme associated to a variety
- **First properties of schemes:** Reduced, integral, noetherian schemes, morphisms of finite type, finite morphisms, closed immersions, fibered product
- **Separated and proper morphisms:** Definition of separated and proper morphisms, criteria, projective morphisms
- **Sheaves of modules:** Definition of an $\mathcal{O}_X$-module, quasi-coherent and coherent sheaves, inverse image, very ample invertible sheaves

- **Divisors:** Weil divisors on schemes regular in codimension one, divisor class group, Cartier divisors, equivalence for locally factorial schemes, Picard group
- **Projective morphisms:** Criteria and characterization of projective morphisms
- **Differentials:** Derivations, module of relative differential forms, sheaves of differentials and connection to nonsingularity
- **Derived functors:** Abelian categories, complexes, derived functors, δ-functors
- **Cohomology of sheaves:** The category of sheaves of $\mathcal{O}_X$-modules has enough injectives
- **Cohomology of a noetherian affine scheme:** Characterization of noetherian affine schemes by cohomology of sheaves
- **Čech cohomology:** Definition, isomorphism with sheaf cohomology for a noetherian separated scheme
- **Cohomology of projective space**

References

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[4] R. Hartshorne, *Algebraic Geometry*, Springer-Verlag, 1977.