Michael Donders
Oral Qualifying Exam
Hill 705
5pm March 4th, 2015

Committee: Professors J. Beck, J. Kahn, M. Saks, and N. Sesum

Primary Topic: Combinatorics And Graph Theory

Combinatorics

- **Enumeration**: bijections, generating functions, binomial and multinomial coefficients, recurrence relations, inclusion-exclusion, Stirling's formula
- **Hypergraphs**: Sperner, LYM inequality, Erdos-Ko-Rado, Kruskal-Katona, Fisher's Inequality (and generalized Fisher), Ray-Chaudhuri–Wilson, Frankl-Wilson, Harper, Baranyai
- **Posets and Lattices:** Dilworth, linear extensions of posets, distributive and geometric lattices, Birkhoff representation theorem, Mobius inversion, Weisner, Dowling-Wilson
- **Ramsey Theory:** Ramsey, infinite Ramsey, Konig tree lemma, probabilistic lower bounds, van der Waerden, statement of Szemeredi
- **Discrepancy:** Beck-Fiala, Roth's 1/4-theorem on arithmetic progressions
- **Probabilistic Method Basics**: Linearity of Expecation, Markov's Inequality, Chernoff bounds, Chebyshev Inequality, statement of Azuma's Inequality, binomial and Poisson distributions
- **Second Moment Method**: thresholds of balanced graphs, clique number
- **Local Lemma**: symmetric and general versions, Ramsey lower bounds, $R(3,k)$

Graph Theory

- **Connectivity:** Greedy algorithm for min weight spanning tree, structure of 2-connected graphs, Mengers theorem, Max-Flow-Min-Cut
- **Hamiltonicity:** Bondy-Chvatal, Posa, Dirac, Ore, Ghouila-Houiri
- **Matchings:** Konig, Hall, Tutte, stable matchings, matching polytopes
- **Planar Graphs:** Euler's formula, Kuratowski, Wagner
- **Coloring:** 5-color theorem, Brooks, Vizing, Thomasson's 5-list-coloring of planar graphs, perfect graphs, Lovasz's proof of weak perfect graph theorem, Galvin's proof of Dinitz conjecture
- **Directed graphs:** Gallai-Haase-Roy-Vitaver, Gallai-Milgram, Mengers theorem for directed graphs
- **Extremal:** Turan, statement of regularity lemma, Erdos-Stone, Chvatal-Rodl-Szemeredi-Trotter
- **Random Graphs:** $G_{n,p}$ vs. $G_{n,M}$, monotone properties, existence of threshold functions, Bollobas-Thomason

Secondary Topic: PDE's

- **First-Order Equations:** Cauchy problem, method of characteristics, characteristic curves and integral surfaces, semi-linear equations, quasi-linear equations, method of Lagrange, weak solutions, conservation laws and jump conditions, general nonlinear equations, Monge cones and characteristic strips

- **General Higher-Order: N**ormal forms, Cauchy-Kovalevski, Hadamard well posed, classification by characteristic, canonical forms, distributions and the Heaviside function, adjoint and weak solutions, convolutions and fundamental solutions
- **Wave Equation:** Domain of dependence and range of influence, Duhamel's Principle, D'alembert, Spherical mean, Euler-Posson-Darboux, Kirchhoff's formula, Hadamard's method of descent, Huygen's Principal, Energy method
- **Laplace Equation:** Separation of variables, Fundamental solution, Green's Functions on half-space and ball, Energy method and Dirichlet's principal, Properties of Harmonic Functions: Mean-Value Property, Maximum Principles, Regularity, Liouville, Harnack's Inequality.

References:

Bollobas, *Modern Graph Theory*, Springer, New York, 1998.

Diestel, *Graph Theory 3rd Edition*, Springer, New York, 2005.

McOwen, *Partial Differential Equations: Methods and Applications 2nd Edition*, Prentice Hall, New Jersey, 2003.