Oral exam syllabus

Michael Weingart

Major topic: Representation Theory

- Essentials of representation theory for finite groups: characters, orthogonality relations, Schur's lemma, complete reducibility

- Lie groups: definitions and examples. The exponential map.

- Explicit constructions of roots, weights, fundamental weights, Weyl groups for classical groups

- Theorem of the highest weight

- Construction of fundamental representations; Cartan products; weights of irreducible representations

- Explicit constructions of irreducible representations of classical groups

- Representations of S_k and Gl_n (duality, parametrization of representations by Young diagrams)

- Clifford algebras and spin representations

- Representations of complex simple groups; representation rings and characters; homogeneous spaces (Borel-Weil theorem, fundamentals of Bruhat decomposition and flag varieties)

- Weyl character and dimension formulas; Freudenthal, Kostant, and Steinberg multiplicity formulas; applications and proofs (at least of WCF).

- Complete reducibility of representations of semisimple algebras (algebraic and analytic proofs)

Minor topic: Lie algebras

- Definitions; solvable and nilpotent algebras; theorems of Engel and Lie

- Cartan's criterion for solvability; Killing form properties, and criterion for semisimplicity

- Root systems from an axiomatic point of view; bases, Weyl chambers, Weyl group

- Classification of Dynkin diagrams and irreducible root systems; affine root systems and generalized Cartan matrices

- Isomorphism and conjugacy theorems: Cartan and Engel subalgebras, conjugacy of Cartan and Borel subalgebras

- Construction of U(g). PBW: statement, proof, and consequences

- Serre's (existence) theorem

- Representations from Humphreys' (purely algebraic) viewpoint

References:

R. Goodman and N. Wallach, *Representations and Invariants of the Classical Groups*

W. Fulton and J. Harris, *Representation Theory*

J. Humphreys, *An Introduction to Lie Algebras and Representation Theory*

W. Fulton, *Young Tableaux*