Oral Qualifying Exam Syllabus

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Major Topic: Lie Algebras and Their Representations

1. Classification of semisimple Lie algebras over $\mathbb{C}$

   - Solvability, nilpotency (Engel's theorem, Lie's theorem, etc.)
   - Structure of semisimple Lie algebras
   - Representation of $\mathfrak{sl}(2, \mathbb{C})$
   - Root system and root space decompostion
   - Classification of semisimple Lie algebras

2. Existence of classical Lie algebras

3. Representation theory of semisimple Lie algebras

   - Universal enveloping algebra and PBW theorem
   - Weights, standard cyclic modules, finite dimensional modules
   - Character formulas

4. Kac-Moody algebras

   - Constructions
   - Representation Theory

5. Construction of affine algebras and extensions of $\mathfrak{g} \otimes \mathbb{C}[t, t^{-1}]$

6. Construction of Basic Modules.

   - Lie Algebra Level
   - Vertex Algebra Construction

Minor Topic: Hopf Algebras

- Definition of Hopf algebra (comodule, bimodule, antipode)
- Coideals and comodules
- Duality – A^0, C^*
- Definition of integrals and smash product
- H-module algebra and coalgebra
- Fundamental theorem of coalgebra
- Coradical filtration
- Quantum groups: $M_q(2)$, $M_q(n)$, $det_q(X)$, $GL_q(2)$, $SL_q(2)$

References

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6. Montgomery, S. *Hopf Algebras and their Actions on Rings.* CBMS Reg. Conf. Series 82, Providence, R.I., 1993.

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