Oral Qualifying Exam Syllabus

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Major Topic: Lie Algebras and Their Representations

1. Classification of semisimple Lie algebras over $\mathbb{C}$
   - Solvability, nilpotency (Engel's theorem, Lie's theorem, etc.)
   - Structure of semisimple Lie algebras
   - Representation of $\mathfrak{sl}(2,\mathbb{C})$
   - Root system and root space decompostion
   - Classification of semisimple Lie algebras
2. Existence of classical Lie algebras
3. Representation theory of semisimple Lie algebras
   - Universal enveloping algebra and PBW theorem
   - Weights, standard cyclic modules, finite dimensional modules
   - Character formulas
4. Kac-Moody algebras
   - Constructions
   - Representation Theory
5. Construction of affine algebras and extensions of $\mathfrak{g} \otimes \mathbb{C}[t, t^{-1}]$
6. Construction of Basic Modules.
   - Lie Algebra Level
   - Vertex Algebra Construction

Minor Topic: Hopf Algebras

- Definition of Hopf algebra (comodule, bimodule, antipode)
- Coideals and comodules
- Duality – A^0, C^*
- Definition of integrals and smash product
- H-module algebra and coalgebra
- Fundamental theorem of coalgebra
- Coradical filtration
- Quantum groups: $M_q(2)$, $M_q(n)$, $det_q(X)$, $GL_q(2)$, $SL_q(2)$

References

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5. Lepowsky, J., Wilson, R. *Construction of the Affine Lie Algebra* $A_1^{(1)}$. Commun. math. Phys. 62, 43–53(1978).

6. Montgomery, S. *Hopf Algebras and their Actions on Rings*. CBMS Reg. Conf. Series 82, Providence, R.I., 1993.

7. Sweedler, M. E. *Hopf Algebras*. Benjamin New York, 1969.