Oral Qualifying Exam Syllabus

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1 Combinatorics

Basic Enumeration: counting arguments, generating functions, binomial coefficients, recurrence relations, inclusion-exclusion principle, Stirling's rule
Set Systems: Sperner's theorem, Erdős-Ko-Rado, Kruskal-Katona, Frankl-Wilson, Harper, Fisher's and generalized Fisher's inequalities, projective geometries, Baranyai
Lattices and Posets: Dilworth, distributive lattices, Birkhoff representation theorem, Dilworth's theorem on dimension of distributive lattices, geometric lattices, Möbius inversion, Weisner, Dowling-Wilson
Correlation Inequalities: Harris-Kleitman, Fortuin-Kasteleyn-Ginibre, Ahlswede-Daykin, application to Shepp's theorems
Ramsey Theory: Ramsey's theorem, infinite Ramsey, König's lemma, probabilistic lower bounds, van der Waerden, Hales-Jewett, statement of Szemerédi's theorem
Discrepancy: Beck-Fiala, Roth's $\frac{1}{4}$-theorem on discrepancy of arithmetic progressions

2 Graph Theory

Matching: König, Hall, Tutte's 1-factor theorem, matching polytope
Connectivity: greedy algorithm for minimum weight spanning tree, structure of 2-connected graphs, Menger, Max-Flow-Min-Cut
Planarity: Euler's formula, Kuratowski's theorem
Coloring: 5 color theorem, Brooks, Vizing, Thomassen's 5-list-coloring of planar graphs, Galvin's proof of Dinitz conjecture, Lovász's proof of weak perfect graph conjecture
Extremal Problems: Turán, statement of regularity lemma, Erdős-Stone, Chvátal-Rödl-Szemerédi-Trotter

3 Probabilistic Methods

Basics: linearity of expectation, alterations, Bonferroni inequalities, Chebyschev's inequality, Chernoff bound

Second Moment Method: general procedure, application to threshold function for having a certain graph as a subgraph
Lovász Local Lemma: symmetric and general versions, applications to hypergraph discrepancy and Latin transversals
Martingales: Azuma's inequality, edge and vertex exposure, application to chromatic number
Poisson Paradigm: Janson inequalities, application to number of triangles in $G_{n,p}$, Brun's sieve, application to number of isolated vertices in $G_{n,p}$
Random Graphs: monotone properties, existence of threshold functions, Bollobás-Thomason, probabilistic refutation of Hajós conjecture
Entropy: basic properties, Shearer's lemma, application to Minc conjecture

4 Probability Theory

Probability Spaces and Random Variables: algebras and σ-algebras, probability spaces, monotone class theorem, independence and product spaces, random variables, distribution functions, expectation, independence of random variables, convergence concepts for random variables, Kolmogorov's 0-1 law
Large Number Laws: weak law of large numbers, Borel-Cantelli lemma, strong law of large numbers, Kolmogorov's three series theorem
Stationary Processes: stationarity, measure-preserving transformations, Birkhoff's ergodic theorem, ergodic theorem for stationary processes
Central Limit Theorem: De Moivre-Laplace theorem, weak convergence and convergence in distribution, characteristic functions, continuity theorem, Lindeberg-Feller central limit theorem
Martingales: conditional expectation, definition of (sub)(super) martingales, stopping times, optional stopping theorems, application to random walks, Doob's martingale inequalities, Doob's upcrossing inequality, uniform integrability, martingale convergence theorems, Lévy's upward convergence theorem