Oral Qualifying Exam Syllabus

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November 20, 2014

1 Combinatorics and Graph Theory

1.1 Combinatorics

Enumeration

Bijections, Generating Functions, Binomial and Multinomial Coefficients, Inclusion-Exclusion, Stirling's Formula, Recurrence Relations

Hypergraphs

Sperner's Theorem, LYM Inequality, Erdős-Ko-Rado, Statement of Kruskal-Katona

Partially-Ordered Sets

Dilworth's Theorem, Linear Extension, Distributive Lattices, Meet-Distributive Lattices, Geometric Lattices, Möbius Inversion, Weisner's Theorem, Dowling-Wilson, Birkhoff Representation Theorem

Ramsey Theorey

Ramsey's Theorem for Graphs and Hypergraphs, Infinite Ramsey, König's Tree Lemma, Upper and Lower Bounds, Probabilistic Lower Bounds, Statement of van der Waerden

Probabilistic Methods

Linearity of Expectation, Markov's Inequality, Chebyshev's Inequality, Chernoff Bounds, Binomial and Poisson Distributions, Method of Alterations, Lovász Local Lemma

Impartial Combinatorial Games

P/N-Positions, Nim-Sum, Misére Play, Games on Graphs, Poset Games, Subtraction Games, Sprague-Grundy Function, Sums of Combinatorial Games

Experimental Mathematics and Applications

Maple Programming

1.2 Graph Theory

Basics

Trees, Bipartite Graphs, Paths, Cycles, Circuits, Walks

Matchings

Hall's Theorem, König's Theorem, Tutte's 1-Factor Theorem, Stable Matchings

Planarity

Euler's Theorem, Kuratowski's Theorem, Wagner's Theorem, Proof that K_5 and $K_{3,3}$ are not planar

Connectivity and Spanning Trees

Structure of 2-Connected Graphs, Structure of 3-Connected Graphs, Menger's Theorem, Max Flow/Min Cut Theorem, Prim's Algorithm, Kruskal's Algorithm, Dijkstra's Algorithm, Bellman-Ford Algorithm

Coloring

Vertex Coloring, Brooks' Theorem, Edge Coloring, Vizing's Theorem, Statement of Weak/Strong Perfect Graph Theorem, Five-Color Theorem

Extremal

Turán's Theorem, Statement of Regularity Lemma, Erdős-Stone Theorem

2 Computational Complexity

P vs NP

Reducibility, Cook-Levin Theorem, Conditions that Imply P$\neq$NP

NP-Complete Problems

SAT, 3-SAT, Independent Set, Clique, 0/1 Integer Programming, Directed Hamiltonian Path

Diagonalization

Undecidability of TM Acceptance Problem (and related problems), Ladner's Theorem

Space Complexity

PSPACE, NPSPACE, L, NL, Savitch's Theorem, PSPACE completeness of TQBF, NL completeness of PATH, NL=coNL

Polynomial Hierarchy

Σ_i, Π_i, Complete Problems, Conditions that lead to PH collapse

Interactive Proofs

Deterministic IP=NP, NP-completeness of GI implies $\Sigma_2 = \Pi_2$, IP=PSPACE