# Oral Qualifying Exam Syllabus

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## 1 Combinatorics and graph theory

### 1.1 Combinatorics

**Basics:** counting arguments, generating functions, binomial coefficients, recurrence relations, inclusion-exclusion principle, Stirling's formula.

**Set systems:** Sperner's theorem, Fisher's and generalized Fisher's inequalities, existence of Steiner systems, Frankl-Wilson theorem, Borsuk conjecture and Kahn-Kalai counterexample, Baranyai's theorem.

**Extremal problems:** Intersecting families, Erdős-Ko-Rado theorem, Harper's theorem, Kruskal-Katona theorem, compression method.

**Lattices:** Geometric and distributive lattices, Birkhoff covering property, Jordan-Dedekind chain condition, Möbius inversion, Weisner's theorem, Dowling-Wilson theorem.

**Correlation inequalities:** Harris/Kleitman theorem, FKG inequality, Four Functions Theorem, application to $xyz$ inequality.

**Discrepancy results:** Erdős-Selfridge theorem, Beck-Fiala theorem, Roth's theorem on 2-D discrepancy, Roth's "second theorem" (discrepancy in arithmetic progressions), properties of positive-density sets.

**Ramsey theory:** Ramsey's theorem for graphs and hypergraphs, König's lemma, infinite Ramsey theorem, general upper bounds, probabilistic lower bounds, stepping-up lemma, van der Waerden's theorem.

*References:* notes for Combinatorics I and II, fall 2001-spring 2002; notes for Combinatorics II, spring 2001; van lint and Wilson, *A Course in Combinatorics.*

## 1.2 Graph theory

**Matchings:** König-Hall theorem, Hungarian algorithm, Tutte's 1-factor theorem, Berge's theorem.

**Connectivity and spanning trees:** basic properties, Menger's theorem, Max-Flow-Min-Cut theorem, Matrix-Tree Theorem, Prüfer codes, minimum-cost spanning trees.

**Planarity:** Euler's formula, Kuratowski's theorem.

**Coloring:** chromatic and edge chromatic numbers, Brooks' theorem, Vizing's theorem, perfect graphs, Lovász perfect graph theorem.

**Extremal problems:** Turán's theorem, Erdős-Stone theorem.

**Regularity lemma:** statement (not proof!) and application to graph Ramsey numbers (CRST result).

**Graph minors:** Wagner's theorem, application to equivalence of four-color theorem and Hadwiger's conjecture for case $r = 5$.

*References:* notes for Graph Theory, spring 2001; Diestel, *Graph Theory.*

## 1.3 Probabilistic methods

**Probability basics:** linearity of expectation, Bonferroni inequalities, binomial and Poisson distributions, conditional probabilities, law of total probability, Chebyshev's inequality, Chernoff bound, coupling and stochastic domination.

**Random graphs:** monotone properties, $G_{n,p}$ versus $G_{n,M}$, existence of threshold functions, relationship between connectedness and having no isolated vertices, probabilistic refutation of Hajós conjecture.

**Second Moment Method:** general procedure, application to threshold function for having a certain graph as a subgraph.

**Lovász Local Lemma:** symmetric and asymmetric versions, applications to hypergraph discrepancy and Latin transversals.

**Martingales:** Azuma's inequality, edge and vertex exposure, application to chromatic number.

**Poisson paradigm:** Janson inequalities, application to number of triangles in $G_{n,p}$; Brun's sieve, application to number of isolated points in $G_{n,p}$.

*References:* notes for Probabilistic Methods in Combinatorics, fall 2001; Alon and Spencer, *The Probabilistic Method,* 2nd ed.

## 2 Computational complexity theory

**Separation theorems:** Gap Theorem, Time and Space Hierarchy Theorems (deterministic and nondeterministic versions), Blum Speedup Theorem.

**Relating circuit and TM complexity:** nonuniform circuit complexity vs. Turing machines with advice functions, definition and bounds for alternating TMs, relationship between uniform circuit families and alternating TMs.

**NC hierarchy:** definitions of $NC^i$, $AC^i$, $SAC^i$, closure of $SAC^i$ under complementation. Branching programs and $NC^1$, nonsolvable group method.

**Reducibility:** polytime many-one, Turing reducibility, stricter circuit reducibility notions ($AC^0$ reducibility etc), randomized reducibility.

**Lower bounds:** simple bounds, polynomial method, Smolensky's theorem. Hastad's switching lemma and application to circuit lower bounds.

**Isolation method:** the Isolation Lemma, application to proof that $NL/poly = UL/poly$.

**Polynomial hierarchy:** unambiguous and probabilistic complexity classes, error types ($n$-sided for $n = 0, 1, 2$), Toda's Theorem.

**Derandomization:** Nisan-Wigderson pseudorandom generator, application to derandomizing BPP.

**Query complexity:** deterministic, nondeterministic, and quantum versions, sensitivity/block sensitivity/certificate complexity bounds, degree bounds, adversary bounds.

**Space-bounded complexity:** Savitch's theorem, closure of NL under complementation.

*References*: notes from Computational Complexity Theory, spring 2002; notes from Quantum Computation, spring 2001; Vollmer, *Introduction to Circuit Complexity*; Hemaaspandra and Ogihara, *The Complexity Theory Companion.*