

# Oral Qualifying Exam Syllabus

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## 1. Stochastic Calculus and Financial Mathematics

- (a) Probability Theory and Stochastic Processes
  - i. Continuity Theorem and Central Limit Theorem
  - ii. Filtrations and Stopping Times
  - iii. Martingales
    - Doob's Martingale Inequality
    - Doob's Optional Stopping Time Theorem
- (b) Stochastic Calculus
  - i. Brownian Motion
    - Scaled Random Walks
    - Construction of Brownian Motion
    - Quadratic Variation
    - Markov Property
    - Reflection Principle
  - ii. Construction of the Ito Integral
  - iii. Ito-Doeblin Formula
  - iv. Multivariable Stochastic Calculus
  - v. Levy's Characterization Theorem
  - vi. Girsanov's Theorem
  - vii. Martingale Representation Theorem
  - viii. Stochastic Differential Equations
    - Existence and Uniqueness Theorems

- Weak and Strong Solutions
- ix. The Markov Property
  - The Markov Property for Ito Diffusions
  - The Strong Markov Property for Ito Diffusions
- (c) Risk Neutral Pricing
  - i. Risk-Neutral Measure
  - ii. Fundamental Theorems Of Asset Pricing
- (d) Connections with Partial Differential Equations
  - i. The Generator of an Ito Diffusion
  - ii. Kolmogorov's Backward Equation and The Resolvent
  - iii. The Feynman-Kac Formula
  - iv. Black-Scholes-Merton Equation
- (e) Options
  - i. European Options
  - ii. Exotic Options
  - iii. American Derivative Securities
- (f) Jump Processes
  - i. Poisson Process
  - ii. Compound Poisson Process
  - iii. Jump Processes and Their Integrals
  - iv. Stochastic Calculus for Jump Processes
  - v. Change of Measure
  - vi. Pricing in a Jump Model

## 2. Partial Differential Equations

- (a) Heat Equation
  - i. Fundamental Solution, Cauchy Problem and Non-Homogeneous Case.
  - ii. Solutions for the Bounded Domain Case
  - iii. Mean-Value Formula
  - iv. Properties of Solutions
    - Strong Maximum Principle, Uniqueness

- Regularity
- Local Estimates for Solutions
- v. Energy Methods
  - Uniqueness
  - Backwards Uniqueness
- (b) Transform Methods
  - i. Fourier Transform
  - ii. Laplace Transform
- (c) Second Order Parabolic Equations
  - i. Definitions
    - Parabolic Equations
    - Weak Solutions
  - ii. Existence of Weak Solutions
    - Galerkin Approximations
    - Energy Estimates
    - Existence and Uniqueness
  - iii. Regularity
  - iv. Maximum Principles
    - Weak Maximum Principle
    - Harnack's Inequality
    - Strong Maximum Principle

**References:**

- [1] Steven E. Shreve: Stochastic Calculus for Finance II, Springer, 2004.
- [2] Bernt Øksendal: Stochastic Differential Equations, Springer-Verlag, 2003.
- [3] Ioannis Karatzas, Steven E. Shreve: Brownian Motion and Stochastic Calculus, Springer-Verlag, 1988.
- [4] D.W. Stroock, S.R.S. Varadhan: Multidimensional Diffusion Processes, Springer-Verlag, 1979.
- [5] Lawrence C. Evans: Partial Differential Equations, AMS Providence, 1998.
- [6] Fritz, John: Partial Differential Equations, Springer-Verlag, 1982.