Oral Qualifying Exam Syllabus

Eduardo Osorio

September 25, 2006

1. Stochastic Calculus and Financial Mathematics

(a) Probability Theory and Stochastic Processes

i. Continuity Theorem and Central Limit Theorem

ii. Filtrations and Stopping Times

iii. Martingales

- Doob's Martingale Inequality
- Doob's Optional Stopping Time Theorem

(b) Stochastic Calculus

i. Brownian Motion

- Scaled Random Walks
- Construction of Brownian Motion
- Quadratic Variation
- Markov Property
- Reflection Principle

ii. Construction of the Ito Integral

iii. Ito-Doeblin Formula

iv. Multivariable Stochastic Calculus

v. Levy's Characterization Theorem

vi. Girsanov's Theorem

vii. Martingale Representation Theorem

viii. Stochastic Differential Equations

- Existence and Uniqueness Theorems

- Weak and Strong Solutions

ix. The Markov Property
- The Markov Property for Ito Diffusions
- The Strong Markov Property for Ito Diffusions

(c) Risk Neutral Pricing
i. Risk-Neutral Measure
ii. Fundamental Theorems Of Asset Pricing

(d) Connections with Partial Differential Equations
i. The Generator of an Ito Diffusion
ii. Kolmogorov's Backward Equation and The Resolvent
iii. The Feynman-Kac Formula
iv. Black-Scholes-Merton Equation

(e) Options
i. European Options
ii. Exotic Options
iii. American Derivative Securities

(f) Jump Processes
i. Poisson Process
ii. Compound Poisson Process
iii. Jump Processes and Their Integrals
iv. Stochastic Calculus for Jump Processes
v. Change of Measure
vi. Pricing in a Jump Model

2. Partial Differential Equations

(a) Heat Equation
i. Fundamental Solution, Cauchy Problem and Non-Homogeneous Case.
ii. Solutions for the Bounded Domain Case
iii. Mean-Value Formula
iv. Properties of Solutions
- Strong Maximum Principle, Uniqueness

- Regularity
- Local Estimates for Solutions

v. Energy Methods
- Uniqueness
- Backwards Uniqueness

(b) Transform Methods

i. Fourier Transform

ii. Laplace Transform

(c) Second Order Parabolic Equations

i. Definitions
- Parabolic Equations
- Weak Solutions

ii. Existence of Weak Solutions
- Galerkin Approximations
- Energy Estimates
- Existence and Uniqueness

iii. Regularity

iv. Maximum Principles
- Weak Maximum Principle
- Harnack's Inequality
- Strong Maximum Principle

References:

[1] Steven E. Shreve: Stochastic Calculus for Finance II, Springer, 2004.

[2] Bernt Øksendal: Stochastic Differential Equations, Springer-Verlag, 2003.

[3] Ioannis Karatzas, Steven E. Shreve: Brownian Motion and Stochastic Calculus, Springer-Verlag, 1988.

[4] D.W. Stroock, S.R.S. Varadhan: Multidimensional Diffusion Processes, Springer-Verlag, 1979.

[5] Lawrence C. Evans: Partial Differential Equations, AMS Providence, 1998.

[6] Fritz, John: Partial Differential Equations, Springer-Verlag, 1982.