DAVID DUNCAN'S ORAL QUALIFYING EXAM SYLLABUS

1. Differential Geometry

1.1. **Differential Topology.** Mayer-Vietoris Theorem for de Rham Cohomology and Singular Homology, Homotopy Invariance of de Rham Cohomology, Smooth Singular Homology, de Rham's Theorem. (Lee, Chs. 15, 16)

1.2. **Morse Theory.** Morse Lemma, Morse Homology, The Morse-Smale Condition, Morse Inequalities. (Milnor, §§1-6)

1.3. **Hodge Theory.** Existence of d^* and $\overline{\partial}^*$, Hodge Theorem, Sobolev Lemma, Rellich Lemma, Gårding's Inequality, Finite Dimensionality of Cohomology, Kodaira-Serre Duality Theorem. (Griffiths and Harris, Ch. 1, §6)

1.4. **Symplectic Geometry.** Canonical Symplectic Structure on the Cotangent Bundle, Lagrangian Submanifolds, Moser Theorems, Darboux's Theorem, Almost Complex Structures, Moment Maps, Symplectic Group Actions, Marsden-Weinstein-Meyer Theorem. (Cannas da Silva, Chs. 1, 2, 3, 6, 7, 12, 13, 14, 21, 22, 23, 26)

1.5. **Bundles and Connections.** Principal G-Bundles, Connections, Gauge Groups. (Cannas da Silva, Ch. 25)

2. Functional Analysis

2.1. **Hilbert Spaces.** The Riesz Lemma. (Reed and Simon, Ch. 2)

2.2. **Banach Spaces.** The Hahn-Banach Theorem, Operations on Banach Spaces, Weak*-Convergence, Open Mapping Theorem, Closed Linear Operators, Closed Graph Theorem, Banach Fixed Point Theorem, Baire Category theorem, Banach-Alaoglu Theorem. (Reed and Simon, Ch. 3)

2.3. **Locally Convex Spaces, Distribution Theory and Sobolev Spaces.** (Reed and Simon, Ch. 5)

2.4. **Bounded Operators.** Adjoints, The Spectrum, Compact Operators, Positive Operators, Self-Adjoint Operators. (Reed and Simon, Ch. 6)

2.5. **The Spectral Theorem.** For Compact and Self-Adjoint Operators. (Reed and Simon, Ch. 7)

3. References

A. Cannas da Silva, *Lectures on Symplectic Geometry*
P. Griffiths, J. Harris, *Principles of Algebraic Geometry*
J. Lee, *Smooth Manifolds*
J. Milnor, *Morse Theory*
M. Reed, B. Simon, *Functional Analysis*