1 Primary Topic- Functional Analysis

1.1 Topological Vector Spaces

1. Types of Topological Vector Spaces

2. Metric spaces and seminorms

3. examples

1.2 Completeness

1. Banach-Steinhaus

2. Open Mapping theorem

3. Closed Graph theorem

1.3 Duality

1. Hahn-Banach theorem

2. weak and weak * topologies

3. examples of duals

4. Compact and integral operators

5. Fredholm alternatives

1.4 Applications

1. Convergence theorems

2. Stone-Weierstrass

3. Haar measure

1.5 Distributions

1. topology of the test functions

2. local representation of distributions

3. PV(1/x), δ function

1.6 Fourier Transform

1. rapidly decreasing functions and tempered distributions

2. Parseval/Plancherel/Inversion on L^2

3. range of Fourier transform

4. Sobolev lemma

1.7 Application to Differential Equations

1. Fundamental solutions

2. Sobolev spaces

3. Elliptic equations and characteristic polynomials

4. regularity theorems

5. solution via Fourier transform

1.8 Bounded operators and Spectral Theorem

1. self-adjoint, unitary, normal operators

2. definition and basic properties of Spectrum

3. Spectral Theorem (Functional Calculus)

1.9 Unbounded operators

1. densely defined operators

2. closed, symmetric, and examples

3. Spectral Theorem (Functional Calculus)

2 Secondary Topic- Differential Geometry

2.1 Manifolds

1. Submanifolds, submersions, immersions

2. homeomorphisms, diffeomorphisms

3. Implicit and Inverse Function theorems

2.2 Tangent Space

1. Tangent space, derivation theorem

2. Vector fields, flows, and the FT ODE

3. Lie Bracket and Derivative

2.3 Bundles

1. bundles, fibers, sections

2. pull back bundle

3. Principle G bundle

4. frame and associated bundle

2.4 exterior algebra

1. cotangent bundle, exterior algebra and derivative

2. closed, exact, de Rham cohomology

3. homotopy invariance

4. insertion, Cartan's formula

5. orientation, orientation of compact manifolds

6. integration, Stoke's theorem

2.5 Covariant derivatives and curvature

1. distributions, vertical, Frobenius' Theorem

2. Connections, covariant derivative, connection 1-form

3. existence theorem

4. parallel transport and lifts

5. Curvature

6. Bianchi identity

7. Metrics, Levi-Civita connection

8. Geodesics

2.6 exp

1. exp, completeness, Hopf-Rinow

2.7   Basic Symplectic Geometry

1. Symplectic forms and manifolds, symplectomorphisms

2. Canonical symplectic structure of cotangent bundle

3. Darboux's theorem, including linear version and Moser's trick

3   References

1. Rudin, W *Functional Analysis*

2. Reed, M and Simon, B, *Functional Analysis*

3. Bott, R and Tu, L, *Differential Forms in Algebraic Topology*

4. Bredon, G, *Topology and Geometry*