Oral Qualifying Exam Syllabus

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1 Graph Theory, Ramsey Theory

Background

Counting, inclusion-exclusion, Stirling Formula, classical finite set systems (Sperner, Erdős-Ko-Rado), elementary applications of probabilistic method.

Flows, Connectivity, and Matching

Max-Flow Min-Cut Theorem; Menger's Theorem; Hall's Theorem; König's Lemma; Tutte's 1-Factor Theorem.

Extremal Graph Theory

Turán's Theorem; Erdős-Stone Theorem; Szemerédi's Regularity Lemma.

Coloring

Brooks' Theorem; Vizing's Theorem; 5-color theorem; Hadwiger's conjecture; Galvin's Theorem; Perfect Graphs, Weak Perfect Graph Theorem; chromatic number of triangle-free graphs.

Planarity

Euler's Formula; Kuratowski's Theorem; Wagner's Theorem.

Hamiltonicity

Bondy Chvátal theorem; Chvátal theorem; Pósa's lemma; Thomason's theorems;

Infinite Graphs

König's infinity lemma, Erdős and de Bruijn's compactness theorem, Unfriendly partition conjecture;

Trees and ends: Jung's theorem; Halin's theorem and the ubiquity conjecture; homogenous and universal graphs (i.e., the Rado graph); The topological end space;

Minors, Trees, and Well-Quasi-Orderings

Quasi- and well-quasi-orderings; the graph minor theorem for trees; tree decompositions and tree width.

Ramsey Theory

Erdős-Szekeres Ramsey theoerems; Infinite Ramsey numbers; Schur's Theorem; van der Waerden's theorem; Hales–Jewett Theorem and Shelah's upper bound.

2 Graphs and Groups

Automorphisms

Automorphism groups of finite trees; automorphism groups of tournaments; universality theorems: graphs (Frucht), trivalent graphs (Frucht),

The graph isomorphism problem: complexity, relation to similar problems, interactive proof systems; enumeration and Pólya's theorem; "almost all graphs have trivial automorphism group" (Erdős-Rényi).

Graph products and automorphisms.

Vertex-transitive graphs, Cayley graphs

Cayley graphs, Schreier coset graphs, Free groups, Reidemeister-Schreier theorem.

Growth: Gromov's Lemma and local expansion in symmetric graphs; Growth rates of Groups: growth rates of groups H and G when $H \leq G$, etc.

3 Combinatorial Game Theory

Impartial games

Nim-like games; sums of games; Sprague-Grundy theorem. (positive, negative, zero and fuzzy games).

Positional games (Tic-Tac-Toe)

Pairing strategies; Erdős-Selfridge theorem; potential functions; Ramsey games; Games in $\mathbb{R}^2$ (any finite point set is a weak winner, etc.); surplus games, Maker-Breaker, Avoider-Forcer, Picker-Chooser, etc games.