TOPIC I: MANIFOLDS AND MORSE THEORY

Manifolds and Riemannian Geometry
- Whitney Embedding Theorem
- Sard's Theorem
- Brouwer Fixed Point Theorem
- Brouwer Degree
- Vector Fields, Velocity Vector Field
- Index Sum: Poincare-Hopf Theorem, Gauss Mapping
- Framed Cobordisms
- The Pontryagin Construction
- Product Neighborhood Theorem
- The Hopf Theorem
- Homogeneity Lemma
- Focal Points
- First and Second Fundamental Forms
- Principal Curvatures and Radii of Curvature
- Affine Connections:
  - Covariant derivative, Parallel Translation
  - Fundamental Lemma of Riemannian Geometry
- Curvature Tensor of an Affine Connection
- Geodesics, Completeness, Distance, and Hopf-Rinow Theorem
- Vector Bundles
- DeRham Cohomology

Morse Theory
- Morse Lemma
- Homotopy Type in Terms of Critical Values
- Sliding and Cancelling Handles
- Reeb's Theorem
- Lefschetz Theorems

Calculus of Variations Applied to Geodesics
- Path Space of a Smooth Manifold:
  - n-Parameter variations, Variation Vector Fields, Critical Paths
- Energy of a Path
- First and Second Variation Formulas
- Jacobi Fields and Obtaining Jacobi Fields along Geodesics via Variations
- Conjugate Points and their Multiplicity
- Morse Index Theorem
- Homotopy Type in Terms of Geodesics
- Topology of the Full Path Space:
  - Fundamental Theorem of Morse Theory, Path Space of the Sphere
- Existence of Non-Conjugate Points
- Some Relations Between Topology and Curvature:
  - Sectional Curvature, Manifolds with Negative Sectional Curvature
  - Cartan's Theorem
  - Ricci Tensor and Myer's Theorem
  - Path Space as CW-Complex

TOPIC II: COARSE GEOMETRY

A. Coarse Geometry on Metric Spaces
   1. Coarse Maps on Metric Spaces
   2. Word Metric and Cayley Graphs

B. General Coarse Structures
   1. Abstract Coarse Structure
   2. Bounded, Proper, and Coarse Sets and Structures
   3. Topological Coarse Structure Associated to a Compactification
   4. Higson Compactification and Corona
   5. Metrizable Coarse Structures
   6. Growth Type of Functions and Bounded Geometry Coarse Spaces
   7. Hyperbolization

C. Amenability
   1. Amenable Spaces and Groups
   2. Growth
   3. 0-Chains, 1-Chains, Ponzi Schemes, and Folner Sequences

D. Coarse Algebraic Topology
   1. Covers, Nerves, and Metrization
   2. Coarse Cohomology
   3. Coarse Homology

E. Coarse Negative Curvature
   1. Rips Property, Gromov Hyperbolicity, and Gromov Hyperbolic Spaces
   2. Controlling Quasi-Geodesics
      a. Gromov Functions and Gromov Compactifications

F. Limits of Metric Spaces
   1. Hausdorff Distance, Gromov-Hausdorff Distance, Gromov's Compactness Criterion, Gromov-Hausdorff Spaces
   2. Rescaled Limit of Metric Spaces
   3. Ultralimits
   4. Asymptotic Cones

G. The Quasi-Isometry Group of Hyperbolic Space

H. Proof of Mostow Rigidity

TOPIC III: ALGEBRAIC GEOMETRY

A. Varieties
  1. Affine and Projective Varieties
  2. The Zariski Topologies
  3. Morphisms and Rational Maps
  4. Nonsingular Varieties (no completions)

B. Schemes
  1. Sheaves
  2. Affine Schemes and Projective Schemes, Gluing Schemes
  3. Properties of Schemes: Connected, Irreducible, Reduced, Integral, Separated
  4. Morphisms: Morphisms of Finite Type, Open and Closed Immersions, Subschemes, Separated and Proper Morphisms
  7. Dimension
  8. Fibred Product of Schemes
  9. Sheaves of Modules: Coherent and Quasi-Coherent Sheaves, Twisted Sheaves, Invertible Sheaves

C. Divisors: Weil Divisors, Cartier Divisors, Divisors on Nonsingular Curves, Invertible Sheaf Associated to a Divisor

D. Vector Bundles and Line Bundles (Via Shafarevich)

E. Projective Morphisms: Morphisms to Projective Space, Ample Invertible Sheaves, Linear Systems

F. Blow Ups

G. Differentials:
  1. Kaehler Differentials, Sheaves of Relative Differentials
  2. Nonsingular Varieties and Bertini's Theorem
  3. Tangent, Canonical, Normal, and Conormal Sheaves
  4. Geometric Genus

H. Cohomology of Sheaves and of a Noetherian Affine Scheme

I. Cech Cohomology

K. Riemann-Roch Theorem for Curves and Surfaces
  1. Canonical Divisor, Geometric and Arithmetic Genus, Intersection Multiplicity
  2. Adjunction Formula

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  Morse Theory Secondary Source: *Introduction to Morse Theory*, Matsumoto
  Morse Theory Additional Source: Professor Hoeffer's Course
2. Coarse Geometry
  Primary Source: *Coarse Geometr*y, John Roe
3. Algebraic Geometry
  Primary Source: *Algebraic Geometry*, Robin Hartshorne
  Secondary Sources: *Basic Algebraic Geometry I, II*, Igor Shafarevich,
    The Geometry of Schemes, Eisenbud and Harris, *Algebraic Geometry*, Harris