Oral Qualifying Exam Syllabus

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Major Topic: Numerical Analysis

I. **Numerical methods for ordinary differential equations**

   i. Derivation and error estimates for one-step methods (e.g., Euler's method)

   ii. Linear Multistep Methods (examples of explicit and implicit methods)

   iii. Consistency, stability, and convergence of multi-step methods

II. **Finite Difference methods for partial differential equations**

   i. Laplace's equation: 5 point difference scheme (Dirichlet and Neumann boundary conditions)

   ii. Discrete maximum principle, existence and uniqueness of the approximate solution

   iii. Consistency, stability, and error estimates

III. **Finite Element methods for elliptic partial differential equations**

   i. Standard variational formulation of second order elliptic boundary value problems

   ii. Ritz-Galerkin approximation schemes and simple error analysis (H^1 and L^2)

   iii. Construction of finite element subspace: dimension of the spaces, basis functions, degrees of freedom, barycentric coordinates, affine families

   iv. Error estimates for polynomial and piecewise polynomial interpolation; Bramble-Hilbert lemma

   v. A posteriori error estimates

IV. Finite Difference and Finite Element methods for the heat equation

   i. Explicit and implicit Euler, Crank-Nicolson

   ii. Analysis of the semi-discrete finite element method

V. Linear Algebraic Solvers

i. Jacobi, Gauss-Seidel, SOR
ii. Conjugate-Gradient
iii. Preconditioning
iv. Multigrid

Minor Topic: Partial Differential Equations

i. Laplace's equation: Fundamental solutions, mean value formulas, properties of harmonic functions, Green's functions, energy methods
ii. Heat equation: Fundamental solutions, mean value formulas, properties of solutions, energy methods
iii. Wave equation: Solution by spherical means, nonhomogeneous problems, energy methods
iv. Sobolev spaces: Hölder spaces, weak derivatives, definition and elementary properties of Sobolev spaces (e.g., trace theorems/embedding theorems).
v. Second-Order Elliptic Equations
   a. existence of weak solutions; Lax-Milgram, energy estimates, Fredholm alternative
   b. Weak and strong maximum principles
   c. Harnack's inequality
   d. Regularity theory

References

[1] R. Falk, *Lecture notes of Numerical Solution of PDE (Math575), Spring 2008*

[2] R. Falk, *Lecture notes of Numerical Analysis (Math573), Fall 2009*

[3] R. Falk, *Lecture notes of Numerical Analysis (Math574), Fall 2010*

[4] D. N. Arnold, *Instructor's notes: Numerical Analysis of Differential Equations*, MATH 8445, University of Minnesota, Fall 2009.

[5] L.C. Evans, *Partial Differential Equations*, AMS, 1991.