Oral Qualifying Exam Syllabus

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1. Algebraic Topology

The Fundamental Group
The Seifert-Van Kampen Theorem
Covering Spaces
Lifting criterion/ Lifting properties
Deck Transformation group

Singular Homology
First homology group as the abelianization of the Fundamental Group
Relative homology
Exact Sequence and Excision
Betti numbers and Euler characteristics
Singular and Cellular Homology
Mayer-Vietoris Sequence

References
Allen Hatcher, Algebraic Topology
James Vick, Homology Theory: An Introduction to Algebraic Topology

2. Riemann Surfaces

Definition of Riemann Surfaces
Maps between Riemann Surfaces
Smooth surfaces

Cotangent spaces and 1-forms
2-forms and integrations
Analytic and meromorphic forms
De Rham cohomology for surfaces

Calculus on Riemann surfaces
Laplace operator and Harmonic functions
The Dirichlet norm

Weyl's lemma
Uniformization theorem
Classification of Riemann surfaces

References

Otto Forster, Lectures on Riemann Surfaces
Simon Donaldson, Notes on Riemann Surfaces

3. Riemannian Geometry

Definition of Riemannian metrics
Riemannian Length and Distance
Space forms/Model spaces
Three models of hyperbolic spaces
The Levi-Civita Connection
Curvature tensors and curvature identities

Reference
John M. Lee, Riemannian Manifolds: An Introduction to Curvature