Oral Exam Syllabus
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1. Combinatorics and Graph Theory

1.1. Combinatorics

Basics: counting arguments, generating functions, binomial coefficients, recurrence relations, reflection principle, inclusion-exclusion, Stirling's formula.
Set Systems: Sperner's Theorem, Kruskal-Katona, Erdős-Ko-Rado, Fisher and generalized Fisher inequality, Frankl-Wilson, Ray-Chaudhuri-Wilson, nonuniform RCW, Steiner systems, isoperimetric problems and Harper's Theorem.
Lattices: geometric and distributive lattices, chains in distributive lattices, linear extensions of posets, the incidence algebra, the Möbius inversion formula, Weisner's Theorem, Dowling-Wilson.
Rational Generating Functions: fundamentals, interplay between rational generating functions, recursive sequences, and closed-form formulae.
Linear Algebra Methods: Combinatorial Nullstellensatz and its applications, inclusion matrices, Chevalley-Warning, Alon-Dubiner, Erdős-Ginzburg-Ziv.
Correlation Inequalities: The Four Functions Theorem, the FKG Inequality, Harris-Kleitman, the XYZ-theorem.
Ramsey Theory: Ramsey's Theorem for graphs and hypergraphs, countable and uncountable Ramsey Theory, König's Lemma, upper and lower bounds, van der Waerden's Theorem and the statements of Roth and Szemerédi.

Sources

Bollobás, *Combinatorics: Set Systems, Hypergraphs, Families of Vectors, and Conditional Probability*
van Lint and Wilson, *Combinatorics*
Babai and Frankl, *Linear Algebra Methods in Combinatorics*
Alon, *Combinatorial Nullstellensatz* (paper)
Jeff Kahn's notes for 640-582, Fall 2004

1.2. Graph Theory

Matching: Hall's theorem, König's theorem, matching algorithms, Gale-Shapley, Augmenting Paths, Tutte's Theorem.
Connectivity and Spanning Trees: Menger's Theorem, Max Flow/Min Cut Theorem, Prim's Algorithm, Kruskal's Algorithm, Dijkstra's Algorithm, Cayley's Theorem, Prüfer Codes.
Planarity: Euler's Formula, Kuratowski, Wagner's Theorem, Tarjan's Algorithm for planarity testing.
Coloring: Chromatic and Edge Chromatic Numbers, List Coloring, Brook's Theorem, Vizing's Theorem, chromatic polynomials and properties, 5-color theorem, perfect graphs, perfect graph theorem.
Extremal Problems: Turán's Theorem, Statement of Regularity Lemma and its application to the Erdős-Stone Theorem.

Sources

Diestel, *Graph Theory*

1.3. Probabilistic Methods

Basics: Linearity of Expecation, Bonferroni Inequalities, common distributions, conditional probability, law of total probability, Chernoff bounds, Chebyshev Inequality, coupling.
Alteration Method: General idea and application to Property B.
Second Moment Method: General procedure and application to threshold functions.
Lovász Local Lemma: Symmetric and general versions, application to Ramsey lower bounds.
Poisson Paradigm: Janson inequalities, application to number of triangles in $G_{n,p}$, Brun's Sieve, application to the number of isolated points.
Martingales: Definitions, Azuma's Inequality, applications to chromatic number.
Random Graphs: Monotone properties, $G_{n,p}$ v. $G_{n,k}$, threshold functions, connectedness.

Sources

Alon and Spencer, *The Probabilistic Method*

Jeff Kahn's notes for 642-591, Fall 2005

2. Computational complexity theory and algorithm analysis

Separation theorems: Gap Theorem, Time and Space Hierarchy Theorems (deterministic and nondeterministic versions), linear speedup theorem, statement of the Blum Speedup Theorem.
P v. NP: Definitions, reducibility, #P, the Cook-Levin Theorem, NP-completeness of INDEPENDENT SET, VERTEX COVER, k-COLORABILITY, PLANAR 3-COLORABILITY, EXACT COVER, KNAPSACK, SUBSET SUM, PARTITION, HAMILTONIAN CYCLE.
Space-bounded complexity: Savitch's theorem, the Immerman-Szeleplcsényi Theorem and implications.
Polynomial hierarchy: Definitions of $\Sigma_i, \Pi_i, \Delta_i$, complete problems, conditions that lead to the collapse of PH.
Circuits: Characterization of P/poly as TMs with advice, The Karp-Lipton Theorem, characterization of PH as DC uniform circuits.
NC hierarchy: Definitions of NC^i, AC^i, SAC^i. Branching programs and NC^1 (Barrington).
Lower bounds: Simple bounds, polynomial method, Smolensky's theorem. Hastad's switching lemma and application to circuit lower bounds.
Probabilistic Proof Systems: Definitions of IP, PCP, and zero-knowledge proofs, Co-NP $\subset$ IP, main ideas behind IP = PSPACE.
Randomization: Definition of RP, BPP and ZPP, BPP $\subseteq \Sigma_2 \cap \Pi_2$, BPP $\subseteq$ P/*poly*, Toda's Theorem.
Derandomization: Nisan-Wigderson pseudorandom generator, application to derandomizing BPP.
Decision Trees: Decision tree complexity, 0- and 1-certificates, certificate complexity, randomized decision tree complexity, sensitivity, block sensitivity.
Communication Complexity: Fooling sets, tiling lowerbound, rank lowerbound.

Sources

Papadimitriou, *Computational Complexity*

Arora, *Computational Complexity: A Modern Approach*

Rudich and Wigderson (editors), *Computational Complexity Theory*