

1. Algebraic Number Theory

- (a) Invariants of number fields: rings of integers, discriminants, orders
- (b) Arithmetic of number fields: splitting of primes, ramification, Frobenius, class groups
- (c) Structure of units in number fields
- (d) Binary quadratic forms
- (e) Kronecker-Weber theorem
- (f) Ideles, adèles
- (g) Chebotarev density theorem, Artin reciprocity, class field theory

2. Analytic Number Theory

- (a) Analytic properties of L-functions
- (b) Primes in arithmetic progressions
- (c) Siegel zero problem
- (d) Prime number theorem and prime number theorem for arithmetic progressions
- (e) Large sieve inequality for multiplicative characters

3. Elliptic Curves

- (a) Elliptic curves over \mathbf{C}
- (b) Elliptic curves over finite fields, Hasse's theorem
- (c) Hasse-Weil L-functions
- (d) Mordell-Weil theorem and descent on elliptic curves
- (e) Complex multiplication

4. Modular Forms

- (a) Modular forms for the full modular group and its congruence subgroups
- (b) Eisenstein series
- (c) Mellin transforms, L-series, converse theorems
- (d) Hecke operators
- (e) Modular forms of half-integer weight