Oral Qualifying Exam Syllabus
Bobby DeMarco
Possible Committee: Profs. J. Beck, J. Kahn (chair), M. Saks, V. Vu

1 Combinatorics

Basic Enumeration: counting arguments, generating functions, binomial coefficients, recurrence relations, inclusion-exclusion principle, Stirling's rule

Set Systems: Sperner's theorem, Erdős-Ko-Rado, Kruskal-Katona, Frankl-Wilson, Harper, Fisher's and generalized Fisher's inequalities, projective geometries, Baranyai, Borsuk's conjecture and Kahn-Kalai counterexample

Lattices and Posets: Dilworth, distributive lattices, Birkhoff representation theorem, Dilworth's theorem on dimension of distributive lattices, geometric lattices, Möbius inversion, Weisner, Dowling-Wilson

Correlation Inequalities: Harris-Kleitman, Fortuin-Kasteleyn-Ginibre, Ahlswede-Daykin, application to Shepp's theorems

Ramsey Theory: Ramsey's theorem, infinite Ramsey, König's lemma, probabilistic lower bounds, van der Waerden, Hales-Jewett, statement of Szemerédi's theorem

Discrepancy: Beck-Fiala, Roth's $\frac{1}{4}$-theorem on discrepancy of arithmetic progressions

2 Graph Theory

Matching: König, Hall, Tutte's 1-factor theorem, matching polytope

Connectivity: greedy algorithm for minimum weight spanning tree, structure of 2-connected graphs, Menger, Max-Flow-Min-Cut

Planarity: Euler's formula, Kuratowski's theorem

Coloring: 5 color theorem, Brooks, Vizing, Thomassen's 5-list-coloring of planar graphs, Galvin's proof of Dinitz conjecture, Lovász's proof of weak perfect graph conjecture

Extremal Problems: Turán, statement of regularity lemma, Erdős-Stone, Chvátal-Rödl-Szemerédi-Trotter

3 Probabilistic Methods

Basics: linearity of expectation, alterations, Bonferroni inequalities, Chebyschev's inequality, Chernoff bound

Second Moment Method: general procedure, application to threshold function for having a certain graph as a subgraph

Lovász Local Lemma: symmetric and general versions, applications to hypergraph discrepancy and Latin transversals

Martingales: Azuma's inequality, edge and vertex exposure, application to chromatic number

Poisson Paradigm: Janson inequalities, application to number of triangles in $G_{n,p}$, Brun's sieve, application to number of isolated vertices in $G_{n,p}$

Random Graphs: monotone properties, existence of threshold functions, Bollobás-Thomason, probabilistic refutation of Hajós conjecture

Entropy: basic properties, Shearer's lemma, application to Minc conjecture

4 Additive Combinatorics

Structure of sumsets and applications basic definitions and results, Ruzsa distance and additive energy, statement of Balog-Szemerédi-Gowers Theorem

Generalized Arithmetic Progressions (GAPs) definitions of proper and non-proper GAPs, containing lemmas

Discrete Fourier Analysis L^p Theory, Linear Bias, Meshulam's Theorem, Gowers Uniformity Norms, Statement of $U3$ inverse theorem, Bohr sets and Regular Bohr sets, Ubiquity of Regular Bohr sets

Ergodic Theory Ergodic theorems, Khintchine recurrence, Furstenberg's correspondence principle

Littlewood-Offord Combinatorial approach, Halász-type concentration inequality, Fourier-analytic approach