

Syllabus for Oral Examination
Ren Guo

Algebraic Topology

The Fundamental Group
The Seifert-Van Kampen Theorem
Covering Spaces
Lifting properties
Classification of covering spaces
Deck Transformations and group actions

Simplicial homology
Singular homology
Homotopy Invariance
Exact Sequence and Excision
Cellular Homology
Mayer-Vietoris Sequence

Cohomology ring
Künneth formula
Cup and Cap Products
Hopf invariant
Poincare Duality

Reference

Allen Hatcher, Algebraic Topology
James Vick, Homology Theory: An Introduction to Algebraic Topology

Riemannian Geometry

Riemannian metrics

Levi-Civita connection

Parallel translation

Curvature tensor

Sectional curvature, Ricci curvature, Scalar curvature

Geodesic, Exponential map, Gauss lemma

Riemannian manifolds as metric spaces

Hopf-Rinow theorem

First and second variations of arc length

Jacobi fields

Manifolds with constant sectional curvature

Cartan-Hadamard theorem

Bonnet-Myers theorem

Synge theorem

Reference

Karsten Grove, Riemannian Geometry: A Metric Entrance Peter
Petersen, Riemannian Geometry