

## Oral Qualifying Exam Syllabus

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Committee (in alphabetical order): R. Bumby, H. Iwaniec, S. Miller, J. Tunnell.

### 1. Modular Forms

- (a) Modular Forms for the full modular group and its congruence subgroups
- (b) Eisenstein series
- (c) Structure of the ring of modular forms
- (d) Mellin transform
- (e) Hecke operators

### 2. Elliptic Curves

- (a) Elliptic functions and the  $j$ -invariant
- (b) Elliptic curves over the complex field
- (c) Elliptic curves over finite fields, Hasse-Weil Theorem
- (d) Hasse-Weil  $L$ -functions of elliptic curves
- (e) Mordell-Weil Theorem and descent on elliptic curves

### 3. Analytic Number Theory

- (a) Analytic properties of  $L$ -functions and the Riemann zeta functions
- (b) Primes in arithmetic progression
- (c) Siegel zero problem
- (d) Prime number theorem and prime number theorem for arithmetic progressions

### 4. Algebraic Number Theory

- (a) Invariants of number fields: rings of integers, discriminants and orders
- (b) Arithmetic of number fields: splitting of primes, ramification
- (c) Class groups
- (d) Structure of units in number rings