Oral qualifying exam syllabus for Robert McRae
Fall, 2009

Major topic: Vertex operator algebras

1. Definitions and properties.

    (a) Formal calculus.

    (b) The notions of vertex algebra and of vertex operator algebra, and basic properties.

    (c) Rationality, commutativity and associativity; equivalence of various formulations, including "weak" formulations.

    (d) The notions of module and generalized module and basic properties.

2. Representations of vertex (operator) algebras.

    (a) Weak vertex operators.

    (b) The structure of the canonical weak vertex algebra. Local subalgebras and vertex subalgebras of the canonical weak vertex algebra.

    (c) The equivalence between modules and representations.

    (d) General construction theorems for vertex (operator) algebras and modules.

3. Examples of vertex (operator) algebras and modules.

    (a) Vertex (operator) algebras and modules based on the Virasoro algebra.

    (b) Vertex (operator) algebras and modules based on affine Lie algebras.

    (c) Vertex (operator) algebras and modules based on Heisenberg Lie algebras.

    (d) Vertex (operator) algebras and modules on even lattices.

    (e) Vertex operator construction of the affine Lie algebras corresponding to A_n, D_n and E_n.

    (f) Twisted modules for lattice vertex (operator) algebras.

    (g) The Moonshine module–basic structure.

4. Modules for a vertex (operator) algebra.
    (a) Zhu's algebra.
    (b) Opposite vertex operators and contragredient modules.
    (c) Intertwining operators and fusion rules.
    (d) The notion of $P(z)$-tensor product of generalized modules.

Minor topic: Lie algebras

1. Elementary notions and basic theory
    (a) Definitions, examples, representations, modules
    (b) Solvable, nilpotent, simple and semisimple Lie algebras and the Killing form
    (c) Lie's theorem
    (d) Engel's theorem
    (e) Cartan subalgebras
    (f) Cartan's criteria for semisimplicity and solvability
    (g) Semisimple Lie algebras as direct products of simple Lie algebras
    (h) Complete reducibility of modules for semisimple Lie algebras
    (i) Levi decomposition
2. Semisimple Lie algebras and root systems
    (a) Representations of $sl(2)$
    (b) Root space decompostion
    (c) Axiomatics of root systems; simple roots; Weyl group
    (d) Classification
    (e) Construction of root systems

3. Universal enveloping algebras
   (a) Construction of the universal enveloping algebra
   (b) The Poincaré-Birkhoff-Witt theorem
   (c) Free Lie algebras
   (d) Generators and relations, and Serre's theorem

4. Representation theory of Lie algebras
   (a) Ado-Iwasawa theorem
   (b) Standard cyclic modules for semisimple Lie algebras
   (c) Finite-dimensional modules for semisimple Lie algebras

5. Infinite-dimensional Lie algebras
   (a) Kac-Moody Lie algebras
   (b) The Weyl group
   (c) Standard modules

References

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[H] J. Humphreys, Introduction to Lie Algebras and Representation Theory, Second Printing, Revised, Springer-Verlag, 1972.

[J] N. Jacobson, Lie Algebras, Dover, 1979.

[L] J. Lepowsky, Lectures on Kac-Moody Lie Algebras, Université Paris VI, 1978.

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