Oral Qualifying Exam Syllabus

Rebecca Coulson (née Gordon)

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Oral Exam Committee: G. Cherlin, S. Thomas, G. Sargsyan, S. Saraf

1 Primary Topic: Logic

1.1 Model Theory

- Completeness, Compactness, and the Löwenheim-Skolem Theorems [Up and Down, the Tarski-Vaught Test

- Back and Forth, Ehrenfeucht-Fraissé Games

- Atomic, Prime, Saturated, and Homogeneous Models

  - $\aleph_0$-categoricity
  - Omitting types

- Fraissé Constructions

- Quantifier Eliminination

  - For DLO, DAG, ACF, and DCF
  - Hilbert's 17th problem, RCF

- Ultraproducts

  - Ax's Theorem

- $\aleph_1$-categoricity

- Morley Rank, Vaught's 2-cardinal theorem

- Strongly Minimal Sets

- Zilber's Conjecture

- Independence in Simple Theories and Stable Theories

  - Imaginary elements and imaginary algebraic closure
  - Symmetry of independence
  - Axiomatization of independence

1.2 Basic Set Theory

- Ordinals, Cardinals, and Cardinal Arithmetic
- Equivalences of Choice
- König's Lemma
- Baire Category Theorem

1.3 Descriptive Set Theory

- Polish spaces
    - Borel Isomorphism Theorem
    - Borel-generated topologies and the Ramsey-Mackey Theorem
    - Sequential trees
- Borel and Projective Hierarchies
    - Basic definitions, facts including closure properties
    - Existence of universal sets for each, non-collapsing of each
    - Every uncountable Polish space contains an analytic set that is not Borel
    - Equivalence of various definitions of analytic sets
    - Regularity properties: Every analytic subset of a Polish space is measurable, has the Baire property, and has the perfect set property

2 Secondary Topic: Combinatorics

- Binomial theorem and coefficients, multinomial theorem, connection coefficients, generating functions
- Max-Flow Min-Cut Theorem
- Principle of Inclusion-Exclusion
- Linear programming: Strong Duality Theorem, Integer Linear Programs, Complementary Slackness Principle
- Poset properties, including the Sperner property and the LYM property; Sperner's Theorem
- König's Theorem, Hall's Theorem, Dilworth's Theorem
- Graph reconstruction and Lovasz' Theorem
- Erdős-Ko-Rado Theorem
- Probabilistic methods: Chebyshev, Markov, and Chernoff-Hoeffding Inequalities; Lovasz Local Lemma, Alteration Method, Jensen's Inequality
- Lattices: relationship with posets, also with alignments and closure, possible properties