

# ORAL QUALIFYING EXAM SYLLABUS

RACHEL LEVANGER, SPRING 2015

## COMMITTEE

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## 1. MAJOR TOPICS

### 1.1. Algebraic Topology.

- The Fundamental Group
  - Van Kampen theorem
  - Covering Spaces
- Simplicial and Singular Homology
  - Exact sequences
  - Axioms for homology
  - Homology and the Fundamental Group
  - Simplicial approximation
- Simplicial and Singular Cohomology
  - Exact sequences
  - Cup, cap, cross products
  - Künneth formula
  - Poincaré duality
  - Alexander duality
- CW Complexes
  - Cellular homology and cohomology
  - Cellular approximation
  - CW approximation

Sources: Hatcher, *Algebraic Topology*, 2002.

### 1.2. Computational Topology.

- Computational structures
  - Simplicial complexes (abstract, geometric, Čech, Vietoris-Rips) (1)
  - Delaunay and Alpha complexes (1)
  - Cubical complexes (2)
  - Discrete morse complexes (5)

- Computation (for Simplicial/Cubical complexes) of
  - Homology groups (2)
  - Reduction algorithms (2), (3) & (5)
  - Induced maps on homology (2) & (5)
- Persistent homology
  - Computation of persistence diagrams (3)
  - Zig-zag persistence (4)
  - Stability results (1)
- Applications
  - Nonlinear Dynamics (2)
  - Conley Index (2)

Sources:

- (1) Edelsbrunner and Harer, *Computational Topology*, 2009.
- (2) Kaczynski, Mischaikow, and Mrozek, *Computational Homology*, 2004.
- (3) Zomorodian and Carlsson, *Computing persistent homology*, Discrete Comput. Geom. 33 (2005), no. 2, 249-274.
- (4) Carlsson and de Silva. *Zigzag persistence*, Foundations of computational mathematics 10.4 (2010), pp. 367-405.
- (5) Harker, Mischaikow, Mrozek, and Nanda, *Discrete Morse Theoretic Algorithms for Computing Homology of Complexes and Maps*, Foundations of computational mathematics (2013).

## 2. MINOR TOPIC

### 2.1. Probability & Ergodic Theory.

- Probability Spaces and Independence:  $\sigma$ -algebras, Caratheodory's Extension Theorem, infinite probability models.
- Random Variables and Expectation. Convergence of random variables.
- Large number laws for sequences of random variables: Borel-Cantelli Lemma, Markov and Chebychev inequalities, Weak and Strong Law of Large Numbers, Kolmogorov's Three Series Theorem.
- Stationary Processes and the Ergodic Theorem.
- Convergence in distribution and the Central Limit Theorem.
- Conditional expectation.

Sources: Ocone, Lecture Notes from 640:591, 2013.