Syllabus

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1 Enumerative Combinatorics

1.1 Generating Functions

Basics: Formal power series, ordinary generating functions, Dirichlet series
Exponential Formula: exponential generating functions, fundamental theorem of exponential generating functions, Lagrange inversion
Rational Generating Functions: Rational power series, polynomials, quasi-polynomials
References: [Wil94], [Zei08], [Sta97]

1.2 Recurrences

Linear Recurrences: c-finite and p-finite, Homogeneous vs. Non-homogeneous, solving techniques
Non-linear Recurrences: Somos sequences, solving techniques, Laurent phenomenon
References: [GK82]

1.3 Partially Ordered Sets

Basics: Definition of poset, chain, antichain, graded/ranked posets, Hasse diagrams, union of posets, product of posets
Lattices: Meet and join, lattice definition, complemented lattice, distributive lattices, Birkhoff's representation theorem
Other: Incidence algebra, Möbius inversion, Inclusion-Exclusion
References: [Sta97]

1.4 Impartial Combinatorial Games

Theory: P/N-positions, Nim-Sum, misére play, games on graphs, Sprague-Grundy function, sums of combinatorial games
Examples: subtraction games, Nim, coin turning games, green hackenbush
References: [BCG01]

1.5 Experimental mathematics and applications

Maple programming, ansatzes

2 Graph Theory

Basic graph theory: basic graph definitions, trees, bipartite graphs, path and cycles
Matching theory: Hall/König and applications, Tutte's 1-factor theorem, Gallai and Millgram thm
Planarity: Euler's theorem, Kuratowski's theorem, Wagner's theorem
Hamiltonicity: Dirac's theorem, Ore's theorem, Bondy-Chvátal theorem, Hamintonian cycles and degree sequences (Chvátal's theorem).
Graph Algorithms: Kruskal's, Dijkstra's, Max Flow-Min Cut (Ford-Fulkerson)
Coloring: Vertex coloring (Brook's thm), edge coloring (Vizing's thm), statement of weak/strong perfect graph theorem, edge list coloring
References: [Die05]

3 Hypergeometric Functions

Definitions: Basic definition of hypergeometric series for single variable and multivariable, definition in terms of differential equations for single variable.
Summing: Formulas for the sum of a hypergeometric function when x = 1. Explain using Euler integrals, combinatorics, and WZ theory.
Other: q-analogues, difference analogues, A-systems and connections with geometry.

References: [BCG01] Elwyn R. Berlekamp, John H. Conway, Richard K. Guy, *Winning Ways for Your Mathematical Plays*, A. K. Peters, 2001.
[Die05] Reinhard Diestel, *Graph Theory*, Springer-Verlag, Heidelberg, NY, 2005.
[GK82] Daniel H. Greene, Donald E. Knuth, *Mathematics for the Analysis of Algorithms*, Birkhauser, Boston, MA, 1982.
[Sta97] Richard P. Stanley, *Enumerative Combinatorics Volume I*, Cambridge University Press, 1997.
[Wil94] Herbert S. Wilf, *Generatingfunctionology*, Academic Press, Inc., 1994.
[Zei08] Doron Zeilberger, 'Enumerative and Algebraic Combinatorics,' in *Princeton Companion to Mathematics*, (ed.) Timothy Gowers, Princeton University Press, 2008.

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