

Oral Qualifying Exam Syllabus

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1. Major Topic: Math Finance

1. Basic material

Change of measure, Independence lemma

Risk-Neutral measure, Market Price of risk equations

First & Second Fundamental theorem

Feynman-Kac Theorem

Black-Scholes-Merton Equation, put-call parity

2. Jump process model

Poisson process, Compensated Poisson process, Compound Poisson process, Jump process

Ito's formula for jump process

Change of measure for jump process, compound jump process

Compound jump process with Brownian Motion

Pricing a European call in jump model

3. Option

Knock-out barrier option (up and out call)

Lookback option

Asian option

American Option, American perpetual put option

4. Change of Numeraire

Foreign risk neutral measure, pricing product quoted in foreign currency

Exchange rate, Pricing call option on exchange rate

Zero-coupon bonds, T-forward prices, pricing call option under T-forward measure

5. Term-Structure Models

Hull-White interest rate model

CIR model (one factor, two factors)

Vasicek Model (two dimensions)

2. Minor Topic: Probability and Stochastic Calculus

1. Martingales, Stopping times and Filtrations

Indistinguishable, modification, same finite-dimensional distributions

Filtration, right (left) -continuity of filtration, adapted

Measurability and progressively measurability of stochastic process

Stopping time, optional time

Martingale, submartingale, supermartingale, local martingale

Upcrossing inequality (proof for discrete case), Submartingale converge theorem

First & Second submartingale inequality

Doob's maximal inequality

Optional Sampling Theorem

Doob's-Meyer Decomposition (proof for discrete case)

Continuous square-integrable Martingales, quadratic variation

2. Brownian Motion

Brownian Motion, Construction of Brownian Motion, Markov Property

Reflection Principle, Distribution of Brownian Motion and it's running maximum

Distribution of first passage time

Strong law of large number for standard Brownian Motion

3. Stochastic Integration

Simple process, construction of stochastic integral with respect to square integral martingale

Continuous semimartingale, Ito's rule

Martingale characterization of Brownian Motion

Girsanov Theorem, Novikov Condition

4. Stochastic Differential Equations

Strong solutions, existence and strong uniqueness

Gronwall inequality

Weak solutions

Pairwise uniqueness, uniqueness in the sense of probability law

References

- [1] Steven E. Shreve, *Stochastic Calculus for Finance II*
- [2] Ioannis Karatzas, Steven E. Shreve, *Brownian Motion and Stochastic Calculus, second edition*
- [3] Kai Lai Chuang, *A Course in Probability Theory, third edition*
- [4] Ramon Van Handel, *Stochastic Calculus, Filtering and Stochastic Control Lecture Notes*
- [5] Triet Pham, *Lecture Notes of Math Finance II (Math 622)*