

**Topics for oral qualifying exam for Christopher
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Spring, 2011**

Major topic: Vertex operator algebras

1. Definitions and properties
 - (a) Formal calculus
 - (b) The notions of vertex algebra and of vertex operator algebra, and basic properties
 - (c) Rationality, commutativity and associativity; equivalence of various formulations, including “weak” formulations
2. Representations of vertex (operator) algebras
 - (a) The notion of module and basic properties
 - (b) Weak vertex operators
 - (c) The structure of the canonical weak vertex algebra; Local subalgebras and vertex subalgebras of the canonical weak vertex algebra
 - (d) The equivalence between modules and representations
 - (e) General construction theorems for vertex (operator) algebras and modules
3. Examples of vertex (operator) algebras and modules
 - (a) Vertex (operator) algebras and modules based on the Virasoro algebra
 - (b) Vertex (operator) algebras and modules based on affine Lie algebras
 - (c) Vertex (operator) algebras and modules based on Heisenberg Lie algebras
 - (d) Vertex (operator) algebras and modules on even lattices
 - (e) Vertex operator construction of the affine Lie algebras corresponding to A_n , D_n and E_n
4. The notions and basic properties of twisted modules for a vertex operator algebra by finite order and general automorphisms

Minor topic: Lie algebras

1. Representations of $sl_2(C)$
 - (a) Basic properties
 - (b) Verma modules
 - (c) Irreducible representations
2. Definitions and properties of Kac-Moody Lie algebras
 - (a) Triangular and root space decompositions
 - (b) The relations for a Kac-Moody algebra
 - (c) The invariant bilinear form and the generalized Casimir element
 - (d) The Weyl group
 - (e) Real and imaginary roots, definitions and properties
3. Affine Lie algebras
 - (a) Affine Lie algebras as central extensions of loop algebras
 - (b) Classification of affine Lie algebras, twisted and untwisted
 - (c) Explicit description of root system and Weyl group
4. Representation theory of Kac-Moody algebras
 - (a) Integrable representations of Kac-Moody algebras
 - (b) The category \mathcal{O} , highest-weight modules and Verma modules
 - (c) Formal characters of modules in \mathcal{O}
 - (d) Integrable highest-weight modules over Kac-Moody algebras, the character formula, the numerator formula and the denominator formula
 - (e) Specializations of the character
 - (f) Explicit description for affine Lie algebras

References

- [C] R. Carter, Lie Algebras of Finite and Affine Type, Cambridge University Press, 2005.
- [FHL] I. Frenkel, Y.-Z. Huang and J. Lepowsky, On Axiomatic Approaches to Vertex Operator Algebras and Modules, *Memoirs Amer. Math. Soc.* 104 (1993).
- [FLM] I. Frenkel, J. Lepowsky and A. Meurman, Vertex Operator Algebras and the Monster, Academic Press, 1988.
- [H] Y.-Z. Huang, Generalized twisted modules associated to general automorphisms of a vertex operator algebra, *Communications in Mathematical Physics*, Volume 298, Issue 1, pp.265-292.
- [L] J. Lepowsky, Lectures on Kac-Moody Lie algebras, Université Paris IV, Spring 1978.
- [LL] J. Lepowsky and H. Li, Introduction to Vertex Operator Algebras and Their Representations, *Progress in Math.*, Vol. 227, Birkhäuser, Boston, 2003.