Oral Qualifying Exam Syllabus

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I. Major Topic: Stochastic Calculus and Mathematical Finance

i. Brownian Motion and Martingale:

a. Brownian Motion
b. Scaled Symmetric Random Walk
c. Quadratic Variation
d. Markov Property
e. Reflection Principle
f. First Passage Time Distribution

ii. Stochastic Calculus:

a. Itô's Integral
b. Itô-Doeblin Formula
c. Derivation of Black-Scholes-Merton Equation
d. Put-Call parity
e. Multi-variable Stochastic Calculus

iii. Risk-Neutral Pricing:

a. Change of probability measure
b. Girsanov's Theorem
c. Martingale Representation Theorem
d. Pricing under Risk Neutral Measure
e. Derivation of Black-Scholes-Merton Formula
f. First and Second Fundamental Theorem of Asset Pricing

iv. Connections with Partial Differential Equations:

a. Stochastic Differential Equations
b. The Markov Property of Solutions
c. Feynman-Kac Theorem

v. Exotic Options:

a. Knock-out Barrier Options
b. Lookback Options
c. Asian Options

vi. **Jump Processes:**
   a. Poisson Process and Compensated Poisson Process
   b. Compound Poisson Process
   c. Jump measure, Poisson random measure
   d. Lévy process
   e. Lévy-Itô Decomposition
   f. Lévy-Khinchin formula

II. **Minor Topic: Numerical Analysis**

i. **Polynomial Approximation:**
   a. Lagrange interpolation
   b. Newton interpolation
   c. Piecewise polynomial approximation
   d. Some error results

ii. **Numerical Quadrature**
   a. Midpoint, Trapezoidal and Simpson's rules
   b. Derivation and error formulas
   c. Basic results of Gaussian quadrature formulas

iii. **Numerical methods for ordinary differential equations**
   a. Derivation and error estimates for one-step methods (e.g., Euler's method)
   b. Linear Multistep Methods (examples of explicit and implicit methods)
   c. Consistency, stability, and convergence of multi-step methods

iv. **Finite Difference methods for partial differential equations**
   a. Laplace's equation: 5 point difference scheme
   b. Discrete maximum principle, Existence and uniqueness of the approximate solution
   c. Stability and error estimates

v. **Finite Element methods for elliptic partial differential equations**
   a. Standard variational formulation of second order elliptic boundary value problems
   b. Ritz-Galerkin approximation schemes and simple error analysis
   c. Construction of finite element subspace: dimension of the spaces, basis functions, degrees of freedom, barycentric coordinates
   d. Affine families of finite elements
   e. Error estimates for piecewise linear interpolation

References

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[2] D. Applebaum, *Lévy processes and stochastic calculus* Cambridge University Press, 2004.

[3] R. Cont, P. Tankov, *Financial modelling with jump processes* Chapman & Hall/CRC, c2004.

[4] R. Durrett, *Probability: Theory and Examples*, 3rd Ed.,Duxbury Press.

[5] R. Falk, *Lecture notes of Numerical Solution of PDE (Math575), Spring 2006*

[6] R. Falk, *Lecture notes of Numerical Analysis (Math573), Fall 2005*