Oral Exam Syllabus

1 Lie Groups

- Definition of a Lie group; examples (including classical Lie groups)
- Lie algebras and their relation to Lie groups

  Exponential mapping

  Adjoint and co-adjoint representation
- Representations of compact, connected Lie groups

  Peter-Weyl theorem

  Maximal Tori: existence, uniqueness up to conjugation, Weyl covering theorem, examples for classical groups.

  Weyl group; action on maximal torus and its Lie algebra

  Complexification; roots; positive roots; dominant alcove

  Dynkin diagrams

  Weight spaces, dominant weights

  Highest weight theorem
- Formulae

  Weyl integration formula

  Weyl character formula

  Dimension formula
- Homogeneous vector bundles

  Induced representations

  Frobenius reciprocity

- Borel-Weil theorem

2 Functional Analysis

- Banach spaces

  Examples (L^p spaces, sequence spaces, direct sums, quotients)

  Linear functionals: duals, reflexive spaces, Hahn-Banach theorems

  Baire category theorem, Open Mapping theorem, Closed Graph theorem, Banach-Steinhaus (uniform boundedness) theorem

  Hilbert spaces (polarisation, adjoints, Riesz lemma)

- Topological devices

  Nets

  Compactness (Tychonoff theorem, Urysohn's lemma, Stone-Weierstrass theorem)

  Banach-Alaoglu theorem

- Bounded operator theory

  Adjoints

  Spectrum

  Compact operators

  Fredholm alternative

  Spectral decomposition of compact, self-adjoint operators

- Differential operators and spectral theory

  Schwarz space

  Fourier transform

  Distributions

  Sobolev spaces

References

[1] Bröcker, T. and tom Dieck, T., *Representations of Compact Lie Groups*

[2] Duistermaat, J., and Kolk, J., *Lie Groups*

[3] Reed, M. and Simon, B., *Functional Analysis*