Oral Qualifying Exam Syllabus

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I. Combinatorics, Graph Theory, and the Probabilistic Method

1 Combinatorics

Basic Enumeration: counting arguments, binomial coefficients, inclusion-exclusion principle, Stirling's formula, recurrence relations and generating functions.
Set Systems: Sperner's theorem and LYM inequality, Erdős-Ko-Rado, Kruskal-Katona, Fisher's inequality.
Correlation Inequalities: Harris-Kleitman, Fortuin-Kasteleyn-Ginibre (FKG inequality), Ahlswede-Daykin four functions theorem, application to Shepp's xyz inequality.
Ramsey Theory: Ramsey's theorem, infinite Ramsey, probabilistic lower bounds, van der Waerden, Gallai's theorem, Hales-Jewett.

2 Graph Theory

Matching: König, Hall, Tutte's 1-factor theorem.
Connectivity: Menger, Max-Flow-Min-Cut, Kruskal's algorithm.
Planarity: Euler's formula, Kuratowski's theorem, Wagner's theorem,
Coloring: chromatic number and edge chromatic number, 5 color theorem, Brooks, König edge coloring theorem, Vizing's Theorem.
Extremal Problems: Turán, Erdős-Stone, statement of regularity lemma.

References: Bollobas, *Modern Graph Theory*

3 Probabilistic Methods

Basics: probability spaces and random variables, linearity of expectation, Bonferroni inequalities, Normal, Binomial, and Poisson distributions, conditional probability and law of total probability, Chernoff bound.
Second Moment Method: general procedure, Chebyschev's inequality, application to threshold function for having a certain graph as a subgraph.
Lovász Local Lemma: symmetric and general versions, application to lower bounds of Ramsey numbers.
Poisson Paradigm: Janson inequalities, application to number of triangles in $G_{n,p}$, Brun's sieve.
Random Graphs: monotone properties, existence of threshold functions, connectedness

(Erdős-Renyi), clique number.

References:

Noga Alon and Joel H. Spencer, *The probabilistic method*
Bollobas, *Modern Graph Theory*

II. Additive and Combinatorial Number Theory

Structure of sumsets and applications: basic definitions and results, Ruzsa distance and additive energy, theorem showing $|r_1A - r_2A + r_3B - r_4B|$ is "small" when $|A + B|$ is "small", statement of Balog-Szemerédi-Gowers Theorem.
Geometry of Numbers: basic results for a lattices and convex bodies in $\mathbb{R}^d$, John's Theorem, Ruzsa's Covering Lemma, Volume Packing Lemma, Blichfeldt's lemma, Minkowski's first theorem, Minkowski's second theorem.
Generalized Arithmetic Progressions (GAPs): definitions of proper and non-proper GAPs, containing lemmas.
Freiman's Theorem: Freiman homomorphism, Freiman's Cube Lemma, Ruzsa's lemma, Freiman's Theorem.
Discrete Fourier Analysis and the Littlewood-Offord problem: Definition $X_{\mathbf{v}}^{(\mu)}$ and basic properties of its Fourier representation, Halász-type concentration inequality, basic Littlewood-Offord results, stronger Littlewood-Offord result with Fourier analysis.

References: Terence Tao and Van H. Vu, *Additive Combinatorics*