Topics for oral qualifying exam for Shashank Kanade

Fall, 2011

Major topic: Vertex operator algebras

1. Definitions and properties
   (a) Formal calculus
   (b) The notions of vertex algebra and of vertex operator algebra, and basic properties
   (c) Rationality, commutativity and associativity and the equivalence of various formulations, including "weak" formulations
2. Representations of vertex (operator) algebras
   (a) The notion of module and basic properties
   (b) Weak vertex operators
   (c) The structure of the canonical weak vertex algebra. Local subalgebras and vertex subalgebras of the canonical weak vertex algebra
   (d) Equivalence between modules and representations
   (e) General construction theorems for vertex (operator) algebras and modules
3. Examples of vertex (operator) algebras and modules
   (a) Vertex (operator) algebras and modules based on the Virasoro algebra
   (b) Vertex (operator) algebras and modules based on affine Lie algebras
   (c) Vertex (operator) algebras and modules based on Heisenberg Lie algebras
   (d) Vertex (operator) algebras and modules based on even lattices
   (e) Vertex operator construction of the affine Lie algebras corresponding to A_n, D_n and E_n
4. Tensor category theory for vertex operator algebras
   (a) Contragredient modules for vertex operator algebras
   (b) Strongly graded conformal and Möbius vertex algebras and their generalized modules
   (c) Logarithmic formal calculus and properties of logarithmic intertwining operators
   (d) $P(z)-$intertwining maps and the notion of $P(z)-$tensor product

Minor topic: Lie algebras

1. Poincaré-Birkhoff-Witt theorem.
2. Kac-Moody algebras
   (a) Root space decomposition

   (b) The invariant bilinear form and the generalized Casimir element
   (c) Weyl group
   (d) Real and imaginary roots, definitions and properties

3. Affine Lie algebras
   (a) Classification of affine Lie algebras, twisted and untwisted
   (b) Explicit realization of Affine Lie algebras
   (c) Explicit description of the root system and the Weyl group

4. Representation theory of Kac-Moody algebras
   (a) Integrable representations of Kac-Moody algebras
   (b) The category O, highest-weight modules and Verma modules
   (c) Formal characters of modules in O
   (d) Integrable highest-weight modules, the character formula, the numerator formula and the denominator formula
   (e) Specializations of the character

5. Examples of Chevalley groups given by generators and relations

References

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[HLZ2] Y.-Z. Huang, J. Lepowsky and L. Zhang, Logarithmic tensor category theory, II: Logarithmic formal calculus and properties of logarithmic intertwining operators arXiv:1012.4196.

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[L] J. Lepowsky, Lectures on Kac-Moody Lie algebras, Université Paris VI, 1978.

[LL] J. Lepowsky and H. Li, Introduction to vertex operator algebras and their representations, Birkhäuser, 2004.

[S] R. Steinberg, Lectures on Chevalley groups, Yale university, 1967.