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Oral Qual Exam Syllabus

I. Newtonian Point Mechanics

A. Newton's Equation of Motion
- i) Galilean invariance
- ii) The two body problem for attractive/repulsive 1/r potential
- iii) The Lorentz force law
  - a) Point particle motion in a uniform electric and magnetic field

B. Lagrangian Formulation
- i) Euler-Lagrange equations
- ii) Equivalence to Newton's equation of motion

C. Hamiltonian Formulation
- i) Legendre transformations
- ii) Equivalence to Newton's equation of motion
- iii) Hamiltonian phase space as a symplectic manifold
- iv) Liouville's theorem
- v) The two body problem for attractive/repulsive 1/r potential

D. Hamilton-Jacobi Formulation
- i) Equivalence to Newton's equation of motion
- ii) Canonical transformations
- iii) The two body problem for attractive/repulsive 1/r potential

II. Einsteinian Point Mechanics

A. Changes in regard to classical physics
- i) Poincaré group
- ii) 4 vectors and invariants (proper time, wave operator, etc.)

B. Maxwell's equations for fields given charges/currents
- i) Derivation of wave equation in Lorenz gauge
- ii) Gauge invariance
- iii) Covariant form of Maxwell's equations

III. Quantum Mechanics

A. Schrödinger's Equation
- i) The (non-relativistic) Hydrogen atom as a 1-body problem

IV. PDEs

A. Laplace's Equation
- i) Characteristic surfaces
- ii) Fundamental solutions including derivations
- iii) Green's identities

- iv) Mean value theorem
- v) Green's functions and the Poisson kernel
  - a) Dirichlet problem on the unit ball
- vi) Uniqueness of Dirichlet problem solutions via the maximum principle
- vii) Perron's existence method for the Dirichlet problem

B. The Wave Equation

- i) Characteristic surfaces (light cone)
- ii) C^2 uniqueness via energy
- ii) Fundamental solutions (derivation for n=1,3)
- iii) Duhamel's principle

V. Analysis

A. Hilbert Spaces

- i) Closed subspace decomposition
- ii) Self duality
- iii) Bessel's inequality
- iv) Completeness

B. Elements of Fourier Analysis

- i) Schwarz space as a Fréchet space
- ii) Convolutions
  - a) Young's inequality
  - b) Properties
- iii) Approximations of the identity
  - a) Density of C_c^∞ in L^p
  - b) C^∞ Urysohn Lemma
- iv) Fourier Transform on R^n
  - a) Properties
  - b) Riemann-Lebesgue lemma
  - c) Fourier Inversion
  - d) Plancherel formula

C. The Theory of Distributions

- i) Definitions of convergence of test functions and continuity of functionals
- ii) Derivatives and convolutions of distributions/test functions
- iii) Density of C_c^∞ (U) in D'(U)
- iv) Tempered Distributions
  - a) Examples
  - b) Fourier transform

D. L^2 Sobolev Spaces

- i) Definition of H_s
- ii) Equivalent norms
- iii) Properties, including density of H_s in H_t for $t<s$
- iv) Sobolev embedding theorem for H_s
- v) Rellich's compactness theorem
- vi) Definition of a localized H_s space

vii) The Elliptic regularity theorem for constant coefficients

E. Inequalities

i) Hölder's inequality
ii) Minkowski's inequality for integrals
iii) Sobolev inequality applied to the hydrogen atom (stability of matter)
iv) Heisenberg's inequality (uncertainty principle)