# ORAL QUALIFYING EXAM SYLLABUS
# DESCRIPTIVE SET THEORY; CLASSICAL GROUPS

SCOTT SCHNEIDER

**Classical Descriptive Set Theory**

- Basic Set Theory (Jech 1-8).

Polish and Standard Borel Spaces.

- Basic definitions and examples: $\mathcal{N}$, $\mathcal{C}$, $\mathbb{H}$, $\mathbb{R}$, $[0, 1]$, $[0, 1] \setminus \mathbb{Q}$; embeddings between them; topological properties; $\mathbb{Q}$ not Polish.
- A topological space is Polish if and only if it is homeomorphic to a $G_\delta$ subset of $\mathbb{H}$.
- Every Borel subset of a Polish space is a continuous image of $\mathcal{N}$ and a continuous, injective image of a closed subset of $\mathcal{N}$.
- The Borel Isomorphism Theorem.
- Borel-generated topologies and the Ramsey-Mackey Theorem.
- Sequential trees, including the rank function and Kleene-Brouwer ordering; systems of sets and their associated maps; Souslin, Lusin, and Cantor schemes; the Souslin operation $\mathcal{A}$.
- Cantor-Bendixson analysis on Polish spaces.

The Borel and Projective Hierarchies.

- Basic definitions and facts, including closure properties.
- Existence of universal sets for each; non-collapsing of each. There does not exist a universal Borel set; there does exist a universal analytic set.
- Every uncountable Polish space contains an analytic set that is not Borel.
- The reduction theorem for additive Borel classes and the separation theorem for multiplicative Borel classes.
- Equivalence of various definitions of analytic sets.
- Every coanalytic set is a union of $\aleph_1$ Borel sets.
- Definitions of $\Sigma^1_1$-complete, $\Pi^1_1$-complete; WF is $\Pi^1_1$-complete.
- Regularity properties: every analytic subset of a Polish space is measurable, has the Baire property, and has the perfect set property.
- Strong measure zero and Lusin sets.
- Souslin's Theorem and the First Separation Theorem for analytic sets.

**Geometry of the Classical Groups**.

Chapters 1-5, 7-8 of Taylor.

- Group Actions: primitive groups; Iwasawa's Theorem.
- Affine and Projective Geometry: basic definitions and the fundamental theorems.
- The groups $\Gamma L(V)$, $GL(V)$, $SL(V)$, $P\Gamma L(V)$, $PGL(V)$, and $PSL(V)$.
- Transvections in $SL(V)$.
- The simplicity of $PSL(V)$, using Iwasawa.
- The BN-pair and split BN-pair axioms.
- Flags, chambers, apartments, and buildings.
- The BN-pair of $SL(V)$, including the Weyl group.
- Correlations of $\mathcal{P}(V)$, $\sigma$-semilinear isomorphisms $V \to V^*$, and non-degenerate $\sigma$–sesquilinear forms on $V$.
- The Birkhoff-von Neumann theorem on the classification of forms.
- Symmetric, alternating, hermitian, and quadratic forms.
- Witt's theorem.
- Bases of orthogonal hyperbolic pairs.
- The group $\Gamma L^*(V)$.
- Polar frames and the building of a polarity.
- The groups $Sp(V)$ and $\Gamma Sp(V)$.
- Symplectic bases.
- Symplectic transvections.
- The simplicity of $PSp(V)$, using Iwasawa.
- Symplectic BN-pairs and symplectic buildings.

**References**


- Jech, Thomas. *Set Theory* (Third Edition).
- Srivastava, S.M. *A Course on Borel Sets.*
- Taylor, Donald. *The Geometry of the Classical Groups.*