

Oral Qualifying Exam Syllabus

BY CHRIS STUCCHIO

Asymptotics

Asymptotic Expansion of Integrals

- Integration by parts
- Laplace method and Watson's lemma
- Stationary phase, steepest descent

Summation Procedures

- Euler and Borel summation
- Pade approximation
- Exponential asymptotics, expansions beyond all orders

Perturbation Theory

- Regular and singular perturbations
- Asymptotic matching
- Eigenvalue problems

WKB Theory

- Conditions for validity
- Necessity (boundary layers, global breakdown)
- Turning point problems
- Tunneling
- Multiple scale analysis

Semilinear Schrödinger Equations

Functional Analysis

- Banach spaces, Hilbert spaces, Strichartz (spacetime) spaces, weak topologies
- Sobolev spaces, Gagliardo-Nirenberg, Sobolev dual spaces
- Spectral theorem, semigroups, unbounded operators
- Fredholm Alternative, Banach Alaoglu and other necessary compactness results.

Basic Estimates

- L^p -estimates on $\exp(i\Delta t)$, Strichartz estimates
- Energy estimates
- Pseudoconformal identity and Morawetz estimates

Existence

- Local existence by Kato's method
- Global existence via energy estimates
- Global existence and estimates for small initial data

Behavior

- Local and global regularity, growth of sobolev norms
- Notions of scattering theory
- Repulsive nonlinearities, decay estimates
- Solitons
- Blowup

Stability of Matter

- Thomas-Fermi theory, Lieb's density functional estimate
- Stability of large atoms and molecules
- Ion behavior in the Thomas-Fermi model