Oral Qualifying Examination Syllabus

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1 Combinatorial Number Theory

- Bertrand's postulate; sum of reciprocals of primes; Mertens's theorem.

- Brun's pure sieve; upper bound for the number of twin primes not exceeding x; convergence of the sum of reciprocals of twin primes.

- Schnirelmann density; connections with additive bases; primes as a basis.

- Squares as a basis; sums of three squares; Cauchy's polygonal number theorem.

- Roth's theorem; Van der Waerden's theorem; Schur's theorem; sum-free sets.

- Ordered and unordered partitions; Euler's pentagonal number theorem; Rogers-Ramanujan identities; elementary estimates for the number of unordered partitions.

- Erdős-Fuchs theorem; Sidon sets; B_h-sequences.

2 Diophantine Approximations

- Continued fractions; best approximations; Hurwitz's theorem; quadratic irrationals; Pell's equation.

- Lagrange and Markoff spectra; gaps in the Markoff spectrum; Hall's ray; Markoff's equation.

- Irrationality of $e, \pi, \ln 2$ and $\zeta(3)$; transcendence of e and π; Lindemann-Weierstrass theorem; Gelfond-Schneider theorem.

- Approximability of algebraic numbers; Liouville's theorem and improvements.

- Distribution modulo 1; Weyl's criterion; uniform distribution of $\{n\alpha\}$ for irrational α; uniform distribution of polynomials; normal numbers.

3 Combinatorial Game Theory

- Impartial games; standard and misère Nim; sums of games; Sprague-Grundy theorem.

- Surreal numbers; field structure and total ordering; positive, negative, zero and fuzzy games.

- Positional games; pairing strategy; Erdős-Selfridge theorem; potential functions.

- The digraph of a game; Richardson's theorem; uniqueness of kernel in acyclic digraphs.